2 THEORETICAL FRAMEWORK

This chapter, organised into five sections, reviews the key concepts and constructs that inform this study's design. The first section addresses the concept of student agency, which includes the narrower concept of epistemic agency, and outlines the learning theories pertinent to its forms. The second section focuses on social theories of learning, in particular addressing the knowledge-creation metaphor of learning and Wenger's (1998) communities of practice. The third section considers the conventional pedagogy that this study seeks to transform in order to achieve its aims, and discusses a picture of authority that is useful for describing pedagogy in general and teachers' and students' participation in it. The fourth section lays out the twin theories of knowledge building and shared epistemic agency that underpin this study. In contrast, the fifth section examines other studies that have worked on transforming pedagogies, especially within the context of mathematics education, supporting my claim that what these studies lack is a focus on an innovative pedagogy such as I am developing that supports everyday practice in the mathematics classroom.

2.1 Agency

This section discusses three approaches to agency that have informed the approach that I develop and utilise in this study. They are:

- Bandura's individualistic and calculative perspective on human agency as the "capacity of individual human beings to make choices and to act on these choices in a way that makes a difference in their lives" (Martin, 2004, p. 135).

  This perspective opposed the tradition of behaviourism that viewed human behaviour as determined mechanistically by environmental stimuli.

- Emirbeyer and Mische's "situated agency" (Emirbayer & Mische, 1998, p. 963)), which, drawing on the work of influential 20th-century social philosophers George Herbert Mead, Hans Joas, and John Dewey's, views of agency as a rational and evaluative capacity. In their view, individuals (actors) can respond to changing environments by continually reconstructing their view of the past as they attempt to understand the conditionings of the emergent present, and use this subsequent understanding as the basis upon which to shape and control their future responses. The inherited conception of a "deliberative attitude" (Mead, 1932, p. 76) represents actors as able to actively constitute their environment by selectively controlling their responses to emergent situations and structural factors such as race, culture, gender, and poverty that otherwise constrain their agency.
- Scardamalia's epistemic agency, which identifies the academic sphere as a locus of the knowledge-building practice of learning, and which connects this practice with the general capacity of the human being (Bereiter & Scardamalia, 1998). Epistemic agency "refers to the amount of individual or collective control people have over the whole range of components of knowledge building" (Scardamalia & Bereiter, 2006, p. 106). The word "epistemic" itself, from Ancient Greek *epìstamai* ("to know"), means "relating to knowledge and knowing".

These three approaches to agency are all underpinned by an attention to the social and relational qualities of agency, though the first two have slightly different backgrounds and assumptions from each other. While this study draws on ideas from both Bandura and Emirbayar and Mische's theories, Scardamalia's work on epistemic agency is the primary influence.

2.1.1 Human Agency

This section starts with the work of Albert Bandura – the *locus classicus* of a discussion of agency to which a considerable majority of researchers in the social sciences have referred since its initial dissemination. In a gesture that helped to make him one of the most influential psychologists in modern history, he challenged the then-predominant behaviourist perspective, positing his "Social Cognitive Theory" of learning and development. Bandura dealt with human behaviour and agency in terms of a triadic framework of reciprocity among environmental variables, behaviours, and personal factors such as cognition (Bandura, 1999, p. 156) He later extended this theory to address how people seek to exercise control over their lives by means of the self-regulation of their actions and thoughts (Bandura, 1986). He claimed that much of human behaviour is performed not only to accommodate the preferences of others, but is also "motivated and regulated by internal standards and self-evaluative reactions to [one's] own actions" (Bandura, 1986, p. 20). Moreover, he argues for construing agency as emergent and interactive, claiming that thoughts emerge from neurological processes initiated and sustained by social interactions. From this socio-cognitive perspective, he identifies four moments of human agency that determine the influence of thought on human actions: intentionality (distinguished from the 'intentionality' that is discussed by earlier psychologists Brentano (Fréchette, 2013) and Husserl (Husserl et al., 2019)) , and which continues to be used as a term in cognitive science and philosophy of mind), forethought, self-reactiveness, and self-reflectiveness (Bandura, 2001). Agency, in the first place, can be understood as a characteristic of whosoever carries out their actions intentionally; people are agentic if their actions are intentional. Intentions themselves are

understood as the proactive commitment to bringing about a desired outcome. Furthermore, successful outcomes that are brought about accidentally, even with intention, are not viewed as agentic, given the separation of intention from the decisive action or event. On the other hand, a successfully intentional action may confer agency on a person even if it does not succeed in bringing about the desired outcome. The critical feature of individual agency is the power to generate actions for a given purpose, regardless of whether the outcome of such actions is of benefit or not, or whether it produces the intended consequences. Student A asking an adjacent student, B, for help with a mathematics question is evidence of a student's intention to solve a mathematics problem. The agency emerges in the activity of asking for help, and is present regardless of the outcome of the request – that is, whether or not help is eventually received or whether such help in fact leads to a correct solution.

Forethought extends agency temporally beyond the present moment of intentionality, connecting it with forward-directed planning (Bandura, 2001, p. 7). People anticipate future consequences of their actions and select current actions to bring about future success. An anticipated future success cannot be a source of current motivation and action (i.e. an intention) since it does not exist. However, when individuals represent the consequences of their intended actions cognitively in the present, they become a source of present self-guidance, motivation, and behavioural regulation in anticipation of a projected goal and future outcome. Individuals exercise agency by acting to shape the present to meet a desired future. In this sense, they transcend the constraints of the present. Following through with the previous example, student A asks the questions of student B because they feel that student B's response would help them solve the mathematics problem, a goal which it is in their interest to

achieve. The decision to ask the question requires a degree of forward-planning. In Bandura's terms, forethought is the capacity of student A to be motivated to persevere with seeking to answer the question, as student A can imagine the future benefits that will accrue if they can solve the mathematics problem (p. 7)

Self-reactiveness as a feature of human agency is the ability of the individual to motivate and self-regulate themselves to execute intended actions for a desired outcome. It includes all the sub-functions of self-regulation that link thought to action, such as self-monitoring, self-guidance, and self-correction. Self-reactiveness is an important element for the achievement of intended actions. Thus, in our example, student A is not only a planner and a forethinker; they can also change how they behave in order to encourage student B to give them the answer to their question or to answer further questions. This could involve such strategies as, for example, not giving in to frustration if student B is too slow to respond.

Having solved the problem with the help of student B, student A can also look back and decide on whether their course of action was the right one. This attests to Bandura's final feature of agency, *self-reflectiveness*: the capacity to understand and be aware of one's thoughts and actions and to evaluate their adequacy. In this metacognitive activity, individuals judge the validity of their predictions against the anticipated outcome of their actions. They consider external effects, such as the impact of other people's actions, established practices and beliefs, and the anticipated impact of these factors on their future success. People's beliefs in their capacity to exercise control over their own functioning and over environmental events constitute the final frontier of human agency (p. 10). People act because they believe they can produce effects with their actions. The strength of one's belief in this ability correlates positively with the effort invested in actions.

Bandura's social cognitive theory also recognises the necessity of collective agency in the precipitation of positive effects; indeed, it is clearly the case that individuals work with others to bring about what they cannot accomplish independently. A key ingredient of collective agency is the belief, mutually held by the individuals that make up a group, in their collective power to bring about the desired results; Bandura refers to this as the "belief of collective efficacy" , noting that it consists in the group members' knowledge, intentions, skills, and the "interactive, coordinated, and synergistic flexibles of their transactions" (p. 14), which together determine the group's attainments.

Although Bandura's view of human agency is interactive and relational, it still emphasises the capacities of the individual, even as it recognises collective agency. This individualist view, though proffered in the distinctive context of modern psychology, can be traced back to the conception of agency as personal autonomy leading to individual empowerment and emancipation that was articulated by Immanuel Kant in the 18th century (Biesta & Tedder, 2006, p. 4). An emphasis on the empowerment of the individual student, who can follow a course of action to meet a desired outcome, and persevere and reflect on the achievement of the outcome for future purposes, is relevant to this study. However, students in mainstream education do not learn in isolation, and this study would be limited if it did not progress beyond the individual perspective alone. Schools are institutions with social structures such as rules and regulations, traditional teaching practices, curriculum maps, and school-wide assessments. Educational policy that includes, for instance, the GCSE curriculum also has bearing on the agency of students. Both social structures and educational policies impact students and their agency in emergent classroom situations. They impose competing views of how students should engage with learning and constrain the actions they may want to take to produce an outcome, or cause

students to re-evaluate their thoughts, habits, and beliefs about consequential outcomes. Since this study seeks to challenge the received views of students as passive and constrained, it requires as a framework a conception of human agency that follows Bandura's – in other words, one which considers the subject to be emergent, dynamic, and interactive – while also mitigating the individualist emphasis of the latter's theories, in order to account for the distributed nature of the social and policy-led pressures that weigh on the students' agency.

2.1.2 Situated Agency

In order to do justice to this interplay, I turn to Emirbayer and Mische's sociological conception of agency as *situated* (Emirbayer & Mische, 1998, p. 963). As noted above, Emirbayer and Mische drew on the work George Herbert Mead; they were also influenced by Hans Joas, and John Dewey's work, situating them within the tradition known as American Pragmatism. Pragmatism rejects the mind-matter and rational-normative dichotomies, offering a theory of knowledge that takes as its point of departure the interactions and transactions that take place in nature – itself understood as "a moving whole of interacting parts" (Dewey in Biesta, 2014, p. 36). On a pragmatic view, the experiences of living organisms cannot be separated in thought from their implication in an environment; organisms interactively adapt to their living circumstances, and are constituted by their attunement to ever-changing environmental conditions. Emirbayer and Mische (1998) characterise their approach as 'relational pragmatics', due, on the one hand, to their allegiance with contemporary and classical pragmatism, and, on the other, to their conception of agency as intrinsically relational and social (p. 973). Their view of agency focuses on

actors and their engagement (and disengagement) with the different contexts and environments that constitute their flexible yet structured social universes.

Emirbayer and Mische argue that a conception of agency should neither be limited to considerations of the individual pursuit of interests and needs (as in the Kantian tradition), nor to a view of human actions as totally constrained within cultural and structural contexts (as, for example, in structuralist anthropology (p. 974). Thus, they seek to reconceptualise agency in order to account for the historical and temporal nature of human experience, and to demonstrate how this temporality interacts with structural contexts informed by the past and oriented towards the present and the future. In their view, human actions, through an interplay of habit, imagination, and judgement, reproduce and can also transform the contextual determinations to which they respond. Individuals can orient themselves towards the past, present, or future at any point in time, and change their orientation as they see fit. Applied to our case, the consequence is that students can and do change their relationships with each other and with their contexts.

According to Emirbayer and Mische, agency has three dimensions. In the first dimension, the *iterational* element, individuals can change the dogmatic schemes of action that have developed over time in a society (p. 976). Their agency lies in the capacity for selecting, deciding, locating, and recognising which actions to change, or else contemplating whether to reproduce existing schemas of experience, activities, expectations of others, or situations developed in the past. In other words, it involves participants knowing what to do with existing knowledge and practices. The second dimension is the *projective* element, on which agency is conceptualised as the ability of individuals to reconfigure their current actions to create a desired future. This dimension, that draws parallels to Bandura's forethought and selfreactiveness, is the

creative-reconstructive dimension of agency, where existing cultural practices do not constrain agents' actions, but rather, constitute challenges to which they can respond. They are able to invent new thoughts and actions to bring about a desired future, and do not have to repeat existing actions and established practices; they can develop new responses to problems. They use current knowledge to move beyond themselves and decide where they are now, where they want to be, and how to get there from where they are in the present (p. 984). The third dimension, the *practical–evaluative* element, views agency as the capacity of individuals to exercise contextual judgements. That is, prudent, intelligent, and practical decisions concerning which actions to perform in order to address problematic situations. Here, agency lies in agents' ability to read the present situation and make decisions in real time that may challenge a given state of affairs. This element sees participants increasing in their capacity to bring about change where the consequence of their actions cannot be structured or controlled. In effect, Emirbayer and Mische posit that human agency should be conceptualised as "a temporal embedded process of social engagement informed by the past (in its habitual aspect), but oriented towards the future (as a capacity to imagine alternative possibilities) and towards the present (as the capacity to contextualize past habits and future projects within the contingency of the moment)" (p. 963).

Emirbayer and Mische stress that these three dimensions of agency are analytical distinctions, and that all three can be identified in various degrees within any empirical instance of action. In Figure 2.1 below, I relate these three dimensions of agency with the moments of Bandura's analysis. As an individual proactively commits to bringing about a future action (intentionality), sets in place a course of action to bring about a future result (forethought), motivates themselves to see their

plans through (self-reactiveness), and reflects on the adequacy of their actions (selfreflectiveness), this individual's thoughts and actions are seen to be able to transform or reproduce their structural environment, and can be informed by past habits, oriented towards an imagined future, or based on present judgments.

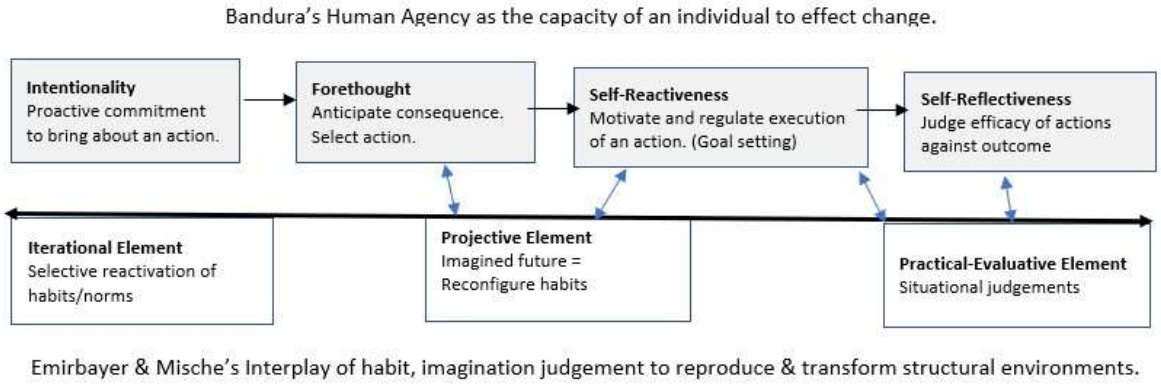

Figure 2.1 – Relating Emirbayer & Mische's (1998) Situational Agency to Bandura's Human Agency (2001)

These two perspectives on agency can be used to analyse the character of student engagement in a school classroom community. Bandura's view is relevant to the extent that it elaborates on the features that underpin students' actions as they strive to produce a desired outcome. His theory offers insights into how students can work interdependently with others to bring about outcomes that they cannot deliver independently. Emirbayer and Mische, on the other hand, contribute the insight that students can bring about desired outcomes by making ad-hoc decisions in the present that could transform the restricted structures in which they are acting. In the context of this study, these decisions might involve deviating from existing habits relating to how students should act in the classroom and finding new ways to develop mathematical knowledge, or indeed simply retaining good habits and traditions. The following section explores how students' decisions and actions can lead to knowledge.

2.1.3 Agency as Epistemic

Scardamalia (2002) argued that the notion of responsibility links human agency, as defined by Bandura (2001), to knowledge, which is the central focus of Scardamalia's conception of epistemic agency. Knowledge arises from choices for which the agent is responsible (Reed, 2001). To 'know', individuals or collectives need to be in control of their actions and have the ability to determine how to apply their will towards concrete forms of action. Individuals or collectives that take responsibility for their learning are aware of what they know or do not know and act on this awareness to advance their knowledge.

The idea that active engagement by participants is required for them to learn or construct knowledge has its roots in Vygotsky's social constructivism (Bereiter, 2002; Scardamalia, 2002; Valsiner & Veer, 2000). Constructivism is a philosophical and psychological perspective on learning that contends that individuals construct or form much of what they learn through their actions and interactions in the world (Packer & Goicoechea, 2000). The sociological applications of constructivism, which emphasise the influence of the social environment on learning, have driven contemporary discussions of agency, its meaning, and its expression in educational environments.

Marlene Scardamalia, a psychologist and educational researcher who is considered one of the pioneers of computer-supported collaborative learning, put forward the notion of epistemic agency (2002) in the context of knowledge-building pedagogy (Scardamalia & Bereiter, 2006). A self-described "deep" constructivist (Scardamalia, 2014), she distinguished between "shallow" constructivist methods such as guided discovery (Brown & Campione, 1994), in which teachers plan what the students are

to discover, and “deep” constructivist methods, in which the highest-level capacities such as planning and the evaluation of learning – which, in our age, are typically accorded only to the teacher — are handed over to the students. Students not only construct their understanding but the whole space of invention, operating as a professional knowledge environment (Scardamalia, 2014, 2:20mins). Emerging from the context of this new learning environment, Scardamalia presents her notion of epistemic agency as the metacognitive ability concerning “goal-setting, motivation, evaluation, and long-range planning” (Scardamalia, 2002, p. 79). In her view, students with epistemic agency assume responsibilities typically left to teachers, and, *pace* Bandura, these students can have collective metacognitive abilities that are different from the mere combination of individual ones. Collectively, students who take responsibility for their learning, form ideas, relate them to others’ ideas, and agree upon an ideal compromise. It is the collective contribution of students that results in and sustains the collective knowledge advancement.

Scardamalia did not provide a clear theoretical account of the concept of epistemic agency, nor describe how it can be identified in an educational setting. I consider the idea of epistemic agency to emerge from her work on collective cognitive responsibility (Scardamalia, 2002). Collective cognitive responsibility exists in groups such as medical teams that carry out knowledge-based work. These groups exhibit qualities such as flexibility, continued learning, collaboration, and rational thinking. Though each member has a specific duty and/or area of expertise, roles are not necessarily fixed. When problems arise, team members can take over from each other without relying on a higher level of authority. The group’s success is distributed across all of the individuals, rather than attributed solely to the leader. In addition to the more tangible and practical aspects, individuals within these teams all

take cognitive responsibility to acquire the knowledge that their activities require and ensure that everyone is adequately knowledgeable. In teams with collective cognitive responsibility, the individuals and the teams are more productive and innovative than those without such responsibility (Scardamalia, 2002). A classroom in which students develop epistemic agency exhibits the characteristics of collective cognitive responsibility. These classrooms act as a community, with collective contributions creating new knowledge and advancing collective knowledge.

2.1.4 Summary

This section has outlined three conceptualisations of agency that emanate from three related perspectives on knowledge, learning, and human development. The first perspective, from which Bandura's (2001) conception of human agency emanates, is Social Cognitivism. This perspective views learning as a reciprocal triad of personal factors, environmental variables, and behaviour. What is in students' minds (thoughts, beliefs) and the teacher's expectations (rules, procedures) influence students' actions and the outcome of these actions. The second perspective is Pragmatism, the perspective of Dewey (1900) and Mead (1932) that represents knowing as based on one's experiences in one's environment; this informs the relational pragmatist viewpoint of Emirbayer & Mische (1998) that represents the relations between ends and means as pre-eminently dynamic, and as unfolding and ongoing processes (Emirbayer, 1997). This view recognises that each student experiences the world uniquely and can react to this experience idiosyncratically as the situation changes for them. Finally, the third perspective of deep Constructivism (Bereiter, 2002; Scardamalia, 2002) argues in favour of students taking responsibility for what they know and do not know and creating knowledge from this process.

These three perspectives of agency are compatible, and dovetail in the notion of epistemic agency, on which taking responsibility for what one knows or does not know transforms individual-situational agency into a new form of agency related to knowledge.

I hold the position that students have the capacity to change and adapt to an innovative pedagogy. While I recognise the agency of the individual students and that of students as a collective as they respond to their classroom learning environment and its pre-existing structures, I lean towards the notion of deep constructivism, appreciating that students can create knowledge as they take responsibility for their learning in a secondary school mathematics classroom. To supplement this perspective, I require a theory that reconciles the social character of learning with this interest in classroom practice.

2.2 Theories of Social Learning

Epistemic agency, as Scardamalia defines it, is, in the classroom context, a quality that sustains the creation of new knowledge by the collective contributions of students who take responsibility for their learning. Having established this, I can identify one goal of this study to be the designing of a pedagogy that supports students in the development of such agency. This innovative pedagogy, elaborated upon in chapter 5, restructures the classroom as an environment in which students can learn as a community. To this end, it draws upon Sfard's (1998) two metaphors of learning, the knowledge-creation metaphor (Paavola et al., 2006), and the social perspective of learning (Wenger, 1998); each of these connects the pedagogical environment with a notion of the community of practice. In the section that follows, I will review these theories to the extent that they underwrite the development of my

own theoretical construct. This review will include an elaboration of the notion of community and power relation.

2.2.1 Metaphors for Collective Learning

Metaphors for learning respond to questions such as who the subject of learning is, the kind of knowledge they should learn, and how they learn it. They reveal certain essential features of learning by asking us to consider it in terms of other behavioural practices. In her article "On Two Metaphors for Learning and the Dangers of Choosing Just One" (1998), the mathematics educator Anna Sfard proposed two primary ways of thinking about how learning occurs: the acquisition and the participation metaphors. The acquisition metaphor depicts knowledge as the capacity of an individual mind, and learning as a process whereby the individual is guided in assimilating or constructing pre-given knowledge. Sfard's participation metaphor, on the other hand, focuses on "knowing" rather than "knowledge". Knowledge does not exist in individuals' minds or in the world, but is situated in the cultural practices of a community (Lave & Wenger, 1991; Rogoff et al., 1998; Wenger, 1998). Learning occurs as individuals participate in and are inculcated into the forms of life that constitute their community. Sfard's presentation of the participation metaphor does not seek to inspire changes in pedagogical practice; rather, her focus is on mastering existing practices. However, thinking her participation metaphor together with the notion of a community of practice as discussed in section 2.2.2, it is clear that participants could, through active negotiation, develop a practice that is both historical and dynamic (Wenger, 1998, p. 53).

Indeed, Paavola, Lipponen, & Hakkarainen (2006) suggest an approach that relies upon but goes beyond the two metaphors mentioned above, highlighting the capacity

for advancing collective knowledge. Their metaphor, that of ‘knowledge creation’ (p. 536), addresses the possibility of innovative learning activities for the creation of knowledge; taking it seriously requires a theory or model of learning that clearly emphasises innovation in relation to learning and knowledge. The knowledgecreation view of learning is connected with the theories of knowledge-building (Scardamalia & Bereiter, 2010) and knowledge creation (Nonaka, 1991) that I discuss in section 4.1 in order to examine what is vital in knowledge communities and innovations in learning, and, ultimately, in order to suggest new approaches to pedagogy.

This *knowledge-creation* approach to learning explicitly builds upon Sfard’s (1998) two metaphors for learning. The acquisition metaphor represents the “monological” view of human cognition and activity, according to which important events happen exclusively within the human mind. In contrast, the participation metaphor emphasises a “dialogical” view of human cognition, whereby important events such as learning occur as the individual interacts with culture, other people, and the surrounding environment. Finally, the knowledge-creation metaphor corresponds to a “trialogical” model (see Figure 2.2); emphasis is placed on the way individuals collaboratively develop shared knowledge objects and artefacts (Paavola et al., 2006, p. 539). In innovative knowledge communities based on the third model, learning occurs during collaborative practices that create shared objects of knowledge.

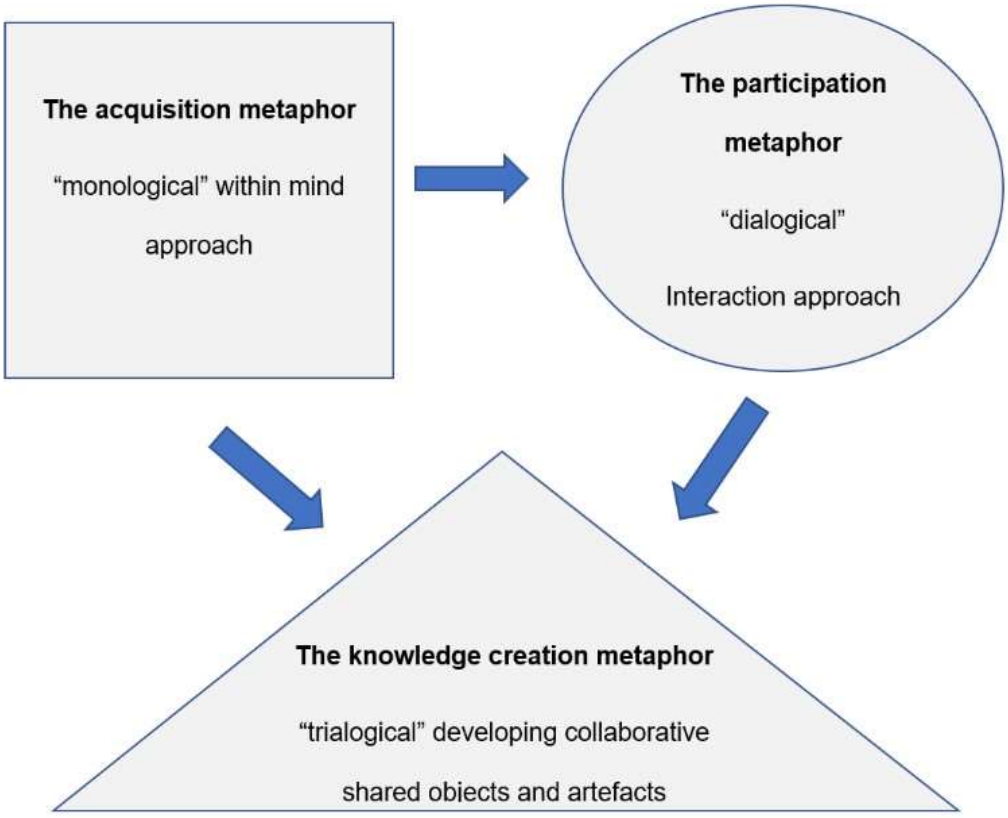

Figure 2.2 – The Three Metaphors of Learning (Paavola & Hakkarainen, 2011, p. 535 - 557)

This proposed innovative pedagogy stands in contrast with a conventional pedagogy (see section 2.3), which relies on the acquisition metaphor, considering the teacher to be in sole control of the transmission of knowledge, and rendering students as passive receivers of this knowledge, having no other role than to store the information received from the teacher. The participation metaphor suggests a pedagogy in which the students are not passive but are required to take an active role in their learning, and points to the idea of students learning as a community. Indeed, in most pedagogies that uphold this metaphor, such as the community of learners model (Brown & Campione, 1996; Rogoff et al., 1998), the classroom is organised as a community with the students working together, all serving as resources for each other and guided by the teacher's leadership.

The knowledge-creation metaphor allows for a further departure from this model, allowing me to describe the classroom and pedagogy that I propose in this study, as

it examines learning in terms of the social structures it creates and the existing processes of collaboration that support innovation and knowledge advancement. This pedagogy views learning as a social process while still recognising the competencies and initiatives of the individuals that make up the community. It focuses on the process of innovation that occurs as people interact, rather than on the contents of individual minds, and brings the dynamics between individuals and environmental structures for creating new knowledge to the forefront. The individuals' initiative feeds the communal effort to innovate, while the social environment feeds the individual's initiative and cognitive development. Constructing shared objects of knowledge requires more than dialogue; it requires the interaction of individuals' contributions and collective contributions in a community learning environment. The proposed pedagogy will focus on students both individually and collectively taking responsibility for their own knowledge creation; the "knowledgecreation" metaphor underpins this pedagogy.

2.2.2 Communities of Practice

My proposal for a new pedagogy based on knowledge creation also requires a sufficiently dynamic conception of the community in which learning takes place. My thinking about community draws on ideas of communities of practice in the work of Etienne Wenger, wherein community relations are of mutual benefit to participants in achieving their shared goals and advancing their mathematical knowledge. In communities of practice, learning is not an individual experience, but rather a social phenomenon that occurs as individuals engage in activities that are essential to the community. Thus, knowledge is competent participation; knowing is the ability to participate in the community's endeavours, and learning involves the transition

towards such competence, changing who a person is. Figure 2.3 below shows the components that characterise participation in Wenger's social theory of learning and knowing, and I will discuss them in turn.

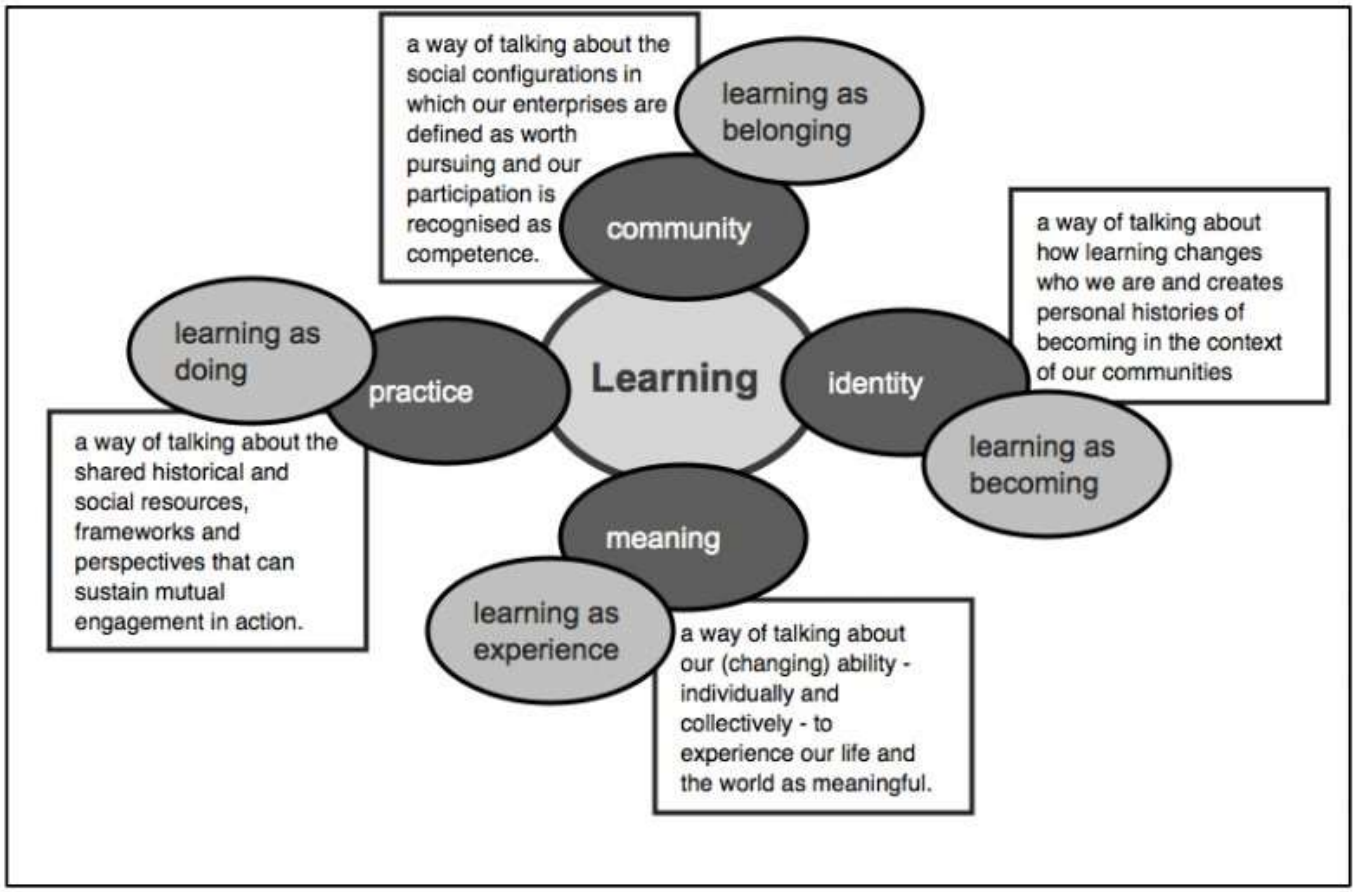

Figure 2.3 – Components of a social theory of learning (Wenger, 1998, p. 5)

2.2.2.1 Learning as Doing

A practice is a way of doing things developed over time by participants of a community of practice to fulfil their purpose of coming together. In a mathematics classroom viewed as a community of practice, the classroom participants, that is, the students and teacher, through their engagement (their doing) over time, develop ways of communicating and behaving that fulfil their aim of learning mathematics. These modalities of communication and behaviour could include students'

knowledge of how to communicate with each other and with the teacher, or of how they access homework and receive feedback. These tacit and explicit classroom practices, negotiated over time, include actions and reifications (Wenger, 1998) that are unique to the participants of that classroom. The term “negotiation” intends “to convey a flavour of continuous interaction, of gradual achievement, and of give and take” (p. 53), emphasising that the participants’ practice is a production of their individuality – of who they are as individuals and who they become as a community.

2.2.2.2 Learning as Experience

Our experience gives meaning to our participation in activity. Wenger used the concept of the “negotiation” of meaning to “characterize the way we as individuals experience the world we are in and how we experience our engagement in it as meaningful” (p. 53). For example, consider the case of students who attend mathematics classes. As they engage in their learning, their activities develop into patterns of action. It is the development of these patterns all over again, lesson after lesson, that constitutes the experience learning mathematics – of what mathematics means to them. The term negotiation is used in the sense of continuous interaction, of the continuous development of meaning through the interactions of participants with their practices.

Reification is a connected term that, in this study, functions to explain the role of material objects in the community of practice. The term refers to the capacity for abstract, distributed, complex ideas to achieve reality as material objects as they assume central functions within a practice. Thus, the curriculum is a reification, as is a lesson plan or a tick in a student’s exercise book. In the classroom, reifications are products of students’ experiences. It is the experience gained through their

participation that gives meaning to what they do. A tick in the book, for instance, conveys to the students that they are correct, knowledgeable, and have given the right answer or solved the problem. A tick is authentic to the students because of the meaning it projects. The meaning of the tick is based on their experience of being in a classroom. Thus, when students say “give me a tick” or “my work was ticked” or “shall I tick it?”, the tick itself is only representative; it is the experience of its meaning, the idea of which it is a reification, that is really circulating. Summarily, reification is “the process of giving form to our experiences by producing objects that congeal this experience into ‘thingness’” (p. 58). The actions and reifications that form the practice of a mathematics classroom give this practice meaning – that is, make visible what learning mathematics is to the participants.

2.2.2.3 Learning as Belonging

The participants of the mathematics classroom negotiate what constitutes competence in the practice of learning mathematics. Competence reflects the actions and reifications that define belonging, that is, being a classroom community member (Wenger, 1998). The community determines competence; it is what the community recognises as competence that defines competence in their community. In some mathematics classrooms, competence is accorded to students who frequently answer questions or put their hands up, or else who complete the set work quickly. Wenger emphasised, however, that belonging to a classroom community requires more than competence alone; it also requires experience of participation. Experience of participation includes mutuality of engagement, establishing relationships with other participants, engaging with them, and responding to their actions and reifications. Accountability for the other participants includes doing what

is required to learn in ways acceptable to the community and its participants. Competent members of a community show their belonging by participating in the practices of the community.

2.2.2.4 Learning as Becoming

As individuals participate in the pedagogy of a mathematics classroom, they build an identity that emerges from the negotiation of what it means to be a member of the classroom community and to engage in its practice of learning mathematics. A participant's identity in the classroom is who they become as a member of the classroom, how they influence the community practice, and how it influences their participation. As participants of a mathematics classroom engage in the practice of learning mathematics, other participants develop relations with them that reify them based on their competence; they are viewing, for example, as good at algebra, at explaining, or at showing their working. Identity involves how we experience our participation and how others project their reifications of our participation on us. Identity can be defined, then, as "a layering of events of participation and reification by which our experiences and its social interpretations inform each other" (p. 151).

A mathematics classroom operating as a community of practice can benefit from the mutual relations inherent in any community with a common purpose. The purpose of advancing their collective mathematics knowledge directs students' participation and practice. As they participate, they negotiate this practice and develop competence as mathematics learners. Their competent participation and participation experience make them belong to the classroom community; as such, participants develop identities and influence the identities of other members of the classroom community. While these ways of viewing learning in a community complement the aims of this

study, Wenger's trajectories of participation (Wenger, 1998, p. 153) that legitimises unequal forms of membership in a community of practice is at odds with this study's view of community. In addition, Wenger has been criticised for his benevolent view of community and for not considering the detrimental impact power/knowledge relations can have on the members of a community (Creese, 2009; Paechter, 2003; Tusting, 2005).

2.2.2.5 The Notion of Community

The notion of community I propose for my mathematics classroom, to meet the aims of this study, draws from the work of the British philosopher John Macmurray. He views community as a mode of unity informed by relationships of the individuals intrinsic worth (McIntosh, 2015, p. 14). These communities are not created or sustained by force but emerge voluntarily and are sustained through friendship. He argued in his book Conditions of Freedom (Macmurray, 1950), that what differentiates these communities from society in general is that they are constitutive of equality and freedom (p.73-74). Akin to friendship, where there is equality of consideration and value, each member of the community has equal value, and their voice counts equally. This does not imply that the individuals are not different in terms of their natural disposition or their capabilities, rather in these communities, the relations between members overrides these differences. The individuals are free to be their authentic self's and express their uniqueness. In essence, equality and freedom are mutually inclusive, they are conditional of each other. Being equal means one can act in accordance with their nature and freedom of expression is made possible amongst equals. Macmurray's view of community suggests human relations in which the individual and the community are interdependent, "we enter

into personal relations with others because it is through them that we can be and become ourselves" (Fielding, 2012, p. 685). The learning is learning to live as a community, both the teacher and the students voluntarily take responsibility for advancement of each other's mathematics knowledge and avoid exercising their freedom in a way that will limit the freedom or the voice of others (McIntosh, 2007, p. 75).

Though Macmurray was calling for education to focus on human fulfilment rather than personal gain, and did not give illustrations of the freedom he described, I can extend this mode of community to a mathematics classroom. The relationships between students and teachers and or between students and students do not depend on their individual functions, that is how they benefit each other, or individual achievement, rather, it is about reciprocal caring for how each other feel in the classroom as they learn together as equals.

I propose a democratic community where the relations of equality and freedom, that exist between students and with the teacher, include participation with a democratic stance (Vinterek, 2010); a classroom where the students trust and respect each other, and have the freedom to take control of how they learn and what they learn, they exhibit a "willingness to listen to others, to speak up and a willingness to give voice to their own thoughts" (p. 373). This proposed classroom community contrasts with the relationships of power that exist in society. Classrooms are microcosms of society, as such, if allowed, hegemony and relations of power, can impacts on the relations between teachers and students and between students and students.

2.2.2.6 Power Relations in Society and the Classroom as a Community

Relationships of power exist in all human interactions and structure human behaviour (Foucault, 1978, p. 96). As individuals are constantly interacting, power is constantly at play in these interactions. Foucault put forward a productive view of power as both positive and negative. He analysed it as something that is capillary, and circulates with individuals as vehicles of power, "not something that is acquired, seized, or shared, something that one holds on to or allows to slip away" (p.94). In effect we all exercise power and are subjected to power by others. In Foucault's view, power exists only when it is put into action. "In effect, what defines a relationship of power is that it is a mode of action which does not act directly and immediately on others. Instead, it acts upon their actions: an action upon an action, on existing actions or on those which may arise in the present or the future" (Foucault, 1982, p. 789). That is to say, the exercise of power directs the conduct of others, it opens possible actions or outcomes, that can be harmful or productive. It also implies a degree of freedom or the possibilities of resistance from others (Foucault, 1980, p. 780), otherwise, there would be no need to direct their conduct. While all individuals or collectives are implicated in power relations, that does not mean that all have equal power. Foucault also posited that power circulates as knowledge and is visible in discourse and discursive practices, such as in the discursive practices of a mathematics class. With this conceptualisation of power in mind, in a classroom community, where teachers and students relate with each other relations of power are at play and could have a positive or negative impact on individuals and the community. This power that circulates will result from the innovative classroom pedagogy, the discourse of schooling that ascribes knowledge hence power to the teacher and from power ascribed to constructs such as race, gender, class, and socioeconomic factors that act to marginalise individuals in

society. An awareness of the workings and source of power are important for this study, if I seek a democratic classroom community as described in the previous section.

2.2.2.6.1 Power Relations in Schools

Some sociologists have claimed that school are structured, designed and organised to mirror the divisions, ranks and hierarchies' existent in society (Giroux, 2011; Giroux & Penna, 1979). The interconnection between ideology, pedagogy and the curriculum acts as a tool to socialises students into society (Bernstein, 2009; Giroux & Penna, 1979). Bourdieu argues that cultural reproduction occurs in schools by normalising what constitutes as knowledge and truth (Bourdieu, 1990). He posits that schools subtly reproduce the power relations that exist in society through mediating the dominant culture that tacitly confirms what being educated means. Michael Apple (2004, pp. 29–30) describes schools as agents of cultural and economic reproduction, maintaining the inequity of society. Hence factors such as race, gender, disability, sexual orientation, socioeconomic status, immigration that disenfranchise sections of our society from participating equitably and democratically (Fraser, 2012; Fraser & Sunkara, 2019; Wallace et al., 2022) can be mirrored in school and in classroom as students and teachers relate with one another.

Educational research (Boaler et al., 2000; Boaler & Greeno, 2000; Gore, 1995; Hargreaves et al., 2021; Smith, 2014; Solomon, 2009c) show that, beyond the inherent discursive practice of school, the power relations at play in society are evident in the mathematics classroom and act to exclude students from full participation. Class, culture and gender caused teachers to position students, in the

mathematics classroom, as competent or not competent thereby, restricting student's access to mathematics knowledge and impacting student's self-belief in their ability to participate in mathematics (Solomon, 2007, 2009c). This positioning acts to limit access to good teaching for low-attaining students (Hargreaves et al., 2021), and limits girls take up of A level mathematics (Smith, 2014).

2.2.2.6.2 Power Circulating between Teacher and Student

The discursive practice of school generally places the teacher by virtue of their knowledge in a position of social dominance in the student teacher relationship, referred to by Bernstein as "control of the social base" , (Bernstein, 2000, p. 30). During student to student interactions, student can copy this teacher attribute and power as social dominance can circulate as mathematics knowledge limiting other students' contributions and mathematic meaning making (Langer-Osuna, 2017). Though the reason why the students mimic the teacher's behaviour could be related to broader institutional norms that focus on competition and comparison rather that community learning (Barron, 2000, p. 432). Understanding the power-relations at work in the classroom is essential if this research seeks to achieve its aims.

As previously stated, power circulates as knowledge, knowledge which Foucault posits is arbitrary. He argued that knowledge is a product of power relations asserting what truth is constructed and kept in place through strategies such as discourse that support and affirm it and exclude and counter other discourses (Foucault, 1978, pp. 100–101). Power operates in the processing of information that selects what is being labelled as fact, that is, in what the curriculum and teachers

allow to be circulated in the classroom. This fact becomes the dominant discourse and other dominated discourses are excluded. Knowledge within schools and in society is carried out and kept in place through a wide range of strategies that affirm and support it such as practice, institutions and hegemony, where those who are dominated by others – such as students, take on board the values and ideologies of the dominant teachers in schools and accept them as their own: this leads to students accepting their position within the hierarchy as natural or for their own good. This internalisation of the dominant discourse by the dominated is the capillary form of power. In this sense, discourses, truth, power and knowledge are intricately linked. This interconnection may give an explanation why the power relations that exist between teachers and students are pervasive such that my attempts to change this dynamic in the classroom may be resisted by the students this study aims to empower.

I find Foucault's work on discourses useful in helping me think about how I know what I know; under what circumstances the information is produced, where it comes from and whose interest it serves. Thus, the discourses of teaching and pedagogy do not hold universal truth but are constructed and held in place by the practices of schooling, it is thus possible to think differently about practices and to trace how what we in schools accept as 'true' is kept in its privileged position.

Consequently, discourses can be seen as a means of resistance as well as a means of oppression. "Discourse transmits and produces power; it reinforces it, but also undermines and exposes it, renders it fragile and makes it possible to thwart it". (Foucault, 1978, pp. 100–101). Though transforming the pedagogy is an act of resistance it is equally an exercise of power because both the students and the

teacher have the freedom to effect change. However, for change to be sustained, the students have to feel that it is purposeful, I have to make the logic of the innovative pedagogy clear to the students, and the aims of the study decipherable (pp. 94–95). To improve students' relationship with and learning of mathematics, it is possible for the students and I to interact on a basis of mutual authority and competence. In exercising power, we can direct each other's conduct towards respect and trust and through enacting an innovative pedagogy build a democratic community – empowering the students to take responsibility for what and of how they learn mathematics.

2.2.3 Summary

At the beginning of this section, I framed a goal of this study, which is to develop a pedagogy that would support students' achievement of epistemic agency. To develop responsibility for their learning requires a pedagogy in which learning is a social endeavour. Thinking about this pedagogy begins with a decision about the metaphor of learning used to describe who the subject of learning is, the kind of knowledge learners should learn, and how learners learn in the pedagogy. The knowledge-creation metaphor (cf. Paavola et al., 2006) provides a way of conceptualising learning in terms of innovative communities of knowledge that does not exclude learning as acquisition or learning as participation; instead, it emphasises how individuals collectively participate to acquire shared knowledge objects and artefacts. This metaphor of learning is of interest to this study as it prepares the context in which epistemic agency can develop and gives form to the goals of the innovative pedagogy that I am developing.

In the second part of this section, I outlined Wenger's social learning theory that discusses how learning can occur in a classroom that operates as a community of

practice, I highlighted the notion of community that will support the aims of this study, considered the power relations at work in society and the relationships of participation that this study's innovative pedagogy aims to develop. However, while the theory outlined four ways of learning in a community – learning as doing, learning as experience, learning as belonging, and learning as becoming – it focuses on knowing rather than knowledge. This social learning theory that focuses of students participating in established practices, therefore, could be viewed as being at odds with a study that focuses on mathematics knowledge and innovative forms of learning within a classroom community. Thus, I hope to draw on the ideas of learning through participating in a community from Wenger (1998), while also moving beyond them by means of the ideas of collective learning from Paavola et al. (2006) in the design of my innovative pedagogy.

2.3 Pedagogy

This section focuses on aspects of pedagogy that will influence this study's innovative pedagogy design. The first sub-section will describe the conventional pedagogy alluded to in section 2.2.1 above. The following two subsections will introduce the constructs of authority and positioning. These two constructs show how the pedagogy can impact the students' experience of and relationship with mathematics in a classroom.

2.3.1 The Conventional Pedagogy

The notion of conventional pedagogy that I introduce here has its roots in my own experience (see section 1.1.3.2), as well as in Paulo Freire's critique of what he

describes in his seminal book, *Pedagogy of the Oppressed*, as the “banking concept of education” (1970, p. 72). In this conception, education takes the form of depositing. The teacher, as the depositor, narrates knowledge to the student who acting as depositories, mechanically receives, memorises, and repeats the information. I consider this teacher-student relationship akin to the acquisition metaphor of learning introduced in section 2.2.1 above. In the banking model, according to which there is an asymmetrical relationship between the teacher and the students, the teacher controls the subject knowledge and its learning as outlined in table 2.1 below.

The Teacher	**The Students**
teaches	are taught
knows everything	know nothing
thinks	are thought about
talks	listen meekly
disciplines	are disciplined
chooses and enforces their choice	Comply
acts	have the illusion of acting through the action of the teacher
Chooses the program content	(who are not consulted) adapt to it
Confuses the authority of knowledge with his or her professional authority, which he or she sets in opposition to the freedom of the student	
is the subject of the learning process.	are mere objects

Table 2.1 – Attitudes and practices of the banking model of education. Quoted from *Pedagogy of the Oppressed* (Freire, 1970, p. 73)

Freire argues that this oppressive pedagogy prevents students' agency from being creative and transformative. He called for an equitable pedagogy based on inquiry in which "knowledge emerges only through invention and re-invention, through the restless, impatient continuing, hopeful inquiry human beings pursue in the world, with the world, and with each other" (Freire, 1970, p. 53). Jacques Rancière, an influential French philosopher who continues to engage with social issues, also criticised the prevalent pedagogy of 1980s France that positioned students as of unequal intelligence to the teacher. He called for an emancipatory pedagogy in which the intelligence of students is recognised and not stultified by what he termed the "explanation logic" (Bingham et al., 2010, p. 3). He posited that schools presuppose students to be ignorant, and present knowledge as needing to be explained by teachers; instead of making students' intelligence equal to that of the teacher, this explanation perpetuates the myths that further explanation is needed, that students are unable to learn without the explanation of the teacher, and that, therefore, they are always of unequal intelligence. Both Rancière and Freire called for a pedagogy of equality, where the polarised view of teacher as knowledgeable and in control, and the students as ignorant and powerless, is replaced by a pedagogy in which students and teachers share authority in the classroom and learn alongside each other.

Although each of these critiques of education had as their contexts different parts of the world and moments in history, I see similarities between the banking model observed by Freire, the inequality of intelligence described by Rancière, and the pedagogy experienced by students in most parts of my school. This pedagogy is clearly based on an unequal relationship between the students and the teacher similar to that outlined in table 2.1 above; I will refer to this as the 'conventional pedagogy', and argue with Boylan (2010), Pratt & Kelly (2007), and Wright et al.

(2020) that this is typical of learning mathematics across England.

Critical mathematics education research, such as that of Gutstein (2006) and Wright (2017), has drawn inspiration from the work of Freire (1970), and has developed mathematics pedagogies with social justice commitments that help students to understand the communities they live in and the ways inequality is contested and produced in the world. This study does not seek to assume a critical perspective on society, though there is an overlap with critical mathematics education in the fact of this study's desire for equality in the authority relations between students and teachers.

An important difference in my aims here, compared with Freire's and Rancière's, is that, while these thinkers aimed at overhauling society to achieve equality and social justice, this study aims to achieve equality in the humbler context of the mathematics classroom, and aims above all at improving the student's relationship with the subject in order to better facilitate their learning.

In the summary of the previous section, I mentioned that the innovative pedagogy based on a knowledge-creation metaphor of learning would aim to have students taking responsibility for their mathematics learning. Taking responsibility for learning requires a pedagogy in which learning is a collective community endeavour, and in which students participate in their learning actively; achieving this state of affairs is an aim of this study. In the conventional pedagogy, wherein the teacher has sole authority, these relations of authority can constrain students' abilities to engage with mathematical ideas and reflect on their learning. In extreme cases, it interferes with their ability to obtain mathematical insights and solve problems in the first place (cf. Amit & Fried, 2005; Brubaker, 2012; Schultz & Oyler, 2006). Thus, in order to avoid

these pitfalls, I turn to a consideration of the phenomenon of classroom authority itself in the preparation of my own pedagogy.

2.3.2 Authority in the Classroom

In an educational context, authority can be defined as a "social relationship where some people are granted the legitimacy to lead, and others agree to follow" (Pace & Hemmings, 2007). It is distinguishable from the form of power, which connotes subjugation of one individual to another's will by some form of coercion (see section 2.2.2.6). Instead, authority involves a relation of obedience and voluntary submission that is quasi-reciprocal rather than coerced. Authority "operates in situations in which a person or group, fulfilling some purpose, project, or need, requires guidance or direction from a source outside himself [sic) or itself" (Benne, 1970, p. 392). Those who lead and those who submit are both relevant to determining the claims to the legitimacy of the authority. Both can determine the extent to which the need for guidance is fulfilled and change the relationship accordingly. Authority requires legitimate claims to competence; otherwise, it becomes a power relationship that involves coercion, a pattern of *over* and *for,* rather than *with* (McNay in Brubaker, 2012).

Authority in education appeals to a value system or normative order that students uphold with their teacher, giving sense to their relationship. Authority cannot be disassociated from the idea of freedom as the students are free to acknowledge the legitimacy of the teacher's authority (Perry et al., 2008). If the students are coerced to accept the teacher's authority, the latter cannot claim their authority as legitimate. The students have the freedom to reject or resist the teacher's authority, but do not do so as they recognise its legitimacy (Goodman, 2010; Peters, 2015). This could be

evidenced in a classroom in which students often moan about the relevance of a particular mathematics topic to their lives – “Miss, how will this help me in real life?” – but nevertheless capitulate to the curriculum requirements stressed by the teacher, knowing of the future benefits of a good mathematics grade. Having said this, in classrooms where the students exercise their freedom to reject or resist teachers’ authority, they could expose themselves to negative consequences, and hence coercion, regardless of whether a given teacher has legitimate claims to competence (Hargreaves, 2017).

As stated in the previous section, in a conventional pedagogy, the teacher is the sole authority. Relevant to this study is the analysis of teacher authority as two interwoven but distinct dimensions of “content” authority and “process” authority (Oyler, 1996a). These dimensions of authority originated in Peters' (1966) view of the teacher as both “an” authority and “in” authority (p. 239-240).

The “content” dimension of authority refers to one who is validated as a knower and viewed as the legitimate possessor of knowledge (i.e., of content). A teacher is an authority carrying out their role as a teacher to teach their subject content. This content authority is referred to in this study as “epistemic” authority (Hargreaves et al., 2018). The use of the term “epistemic” as opposed to “content” is in keeping with Solomon’s (2009a) use of the word epistemic that views mathematics knowledge as open to negotiation and knowers as creative negotiators of mathematics knowledge. Epistemic authority is attributed to the teacher by the definition of the “teacher” role. It presupposes that the teacher has studied to attain the subject knowledge, and is therefore employed by the school. However, the teacher has to demonstrate and establish this authority in the classroom for it to be legitimised by the students (Hargreaves et al., 2018; Wagner & Herbel-Eisenmann, 2014).

On the other hand, a teacher has “process” authority due to an aspect of the prevailing culture: how the knowledge is taught in the classroom in a given society. This process dimension of authority, synonymous with being ‘in’-authority (Peters, 1966), is best understood in terms of the notion of framing (Bernstein, 2000). Framing relates to how knowledge is communicated and the nature of the relations that go along with it. It relates to who is in control of selecting the knowledge to be communicated, the “how” of learning, its sequencing, its pacing, the instructional criteria, the control of the social base, the regulative criteria, and the dominant values of the society that make the communication of knowledge possible (Bernstein, 2000, p. 37). When the teacher is in control, such as in a conventional pedagogy, the framing is said to be “strong”. The teacher has authority over the processes of how the knowledge is communicated to the students. Theoretically, where the students are in control, the framing could be said to be weak; it is important to understand that this is not an evaluation of quality, but of the potency of individuals’ relations to the determination of practice.

As I view students’ active participation in all aspects of their learning as necessary for the aims of this study – to improve student’s relationships and learning of mathematics – this study requires a move away from the conventional pedagogy in which authority is in solely the teacher’s possession in order to achieve its aim. Instead, it calls for a shared authority pedagogy, where the students participate in all aspects of their learning. As Oyler (1996a) notes, this is a more significant move than it would seem: “Sharing authority then is much more than offering activity choices; rather it requires that teachers and students develop and negotiate a common destination or agenda” (p. 23).

2.3.2.1 Shared Authority

The process and epistemic dimensions of authority are not the only ways to construe authority in an educational setting. Various authors identify a range of types of authority (Amit & Fried, 2005; Pace & Hemmings, 2007; Solomon, 2009b; Wagner & Herbel-Eisenmann, 2014). From my reading of Solomon (2009c), I would argue that the process/epistemic distinction points to what one has authority over, while the notion of shared authority addresses whether/how participants distribute authority amongst themselves. Shared authority, also referred to as "revised authority" (Amit & Fried, 2005, p. 151), is the authority characterised by co-participation that involves both the students and the teacher; in this case, the legitimacy of either the students or the teacher's authority comes from mutual interdependency where those involved, such as the teacher and the student, are continually learning and reaching beyond their present relationship to a relationship that "supports independence while acknowledging differences in knowledge, skill and status" (Benne, 1970, p. 401).

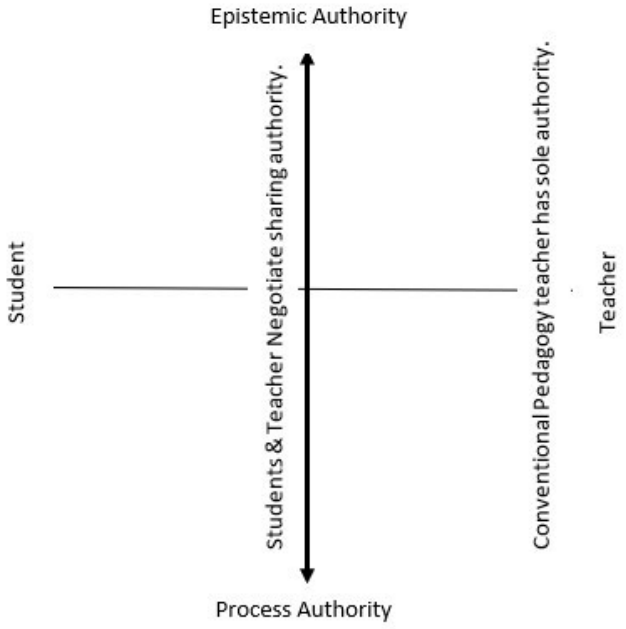

Figure 2.4 – Process/Epistemic dimensions of authority vs teacher/student's authority distribution.

In classrooms with revised authority, students, and the teacher, through their participation, can negotiate how process and epistemic authority is shared (see figure 2.4). The revised authority shifts the focus of authority to negotiation and consent, and renders the relationships upon which authority supervenes as dynamic and fluid (Amit & Fried, 2005). The students do not blindly expect the teacher to be the expert, but see expertise in themselves and in each other (Brubaker, 2012).

Epistemic authority refers to who is viewed as legitimately knowledgeable and process authority refers to how the knowledge is taught in the class. However, teachers also have other relationships to knowledge that support their authority.

Shulman (1986, 2013) coined the term "pedagogical content knowledge" (PCK) to emphasise that a discussion of one's knowledge of a subject is not sufficient to explain what is necessary for teaching. He suggests a trichotomy of categories of content knowledge: subject matter content knowledge, pedagogical content knowledge, and curricular knowledge. In this study, epistemic authority refers to subject matter content knowledge, and the notion of process authority subsumes pedagogical content knowledge and curricular knowledge. As the term suggests, subject matter content knowledge refers to the structure and amount of subject knowledge in the teacher's mind. Pedagogic content knowledge refers to the generic principles of classroom organisation and management, the most useful representations of ideas that make them comprehensible to students' preconceptions, and common misconceptions that students bring with them to topics. Curricular knowledge refers to the full range of topics required for the subject; it includes the sequence of topics, instruction material, and assessment requirements.

From a mathematics perspective, pedagogical content knowledge and curricular knowledge can be conceptualised as mathematics knowledge for teaching (MKT) (J. Silverman & Thompson, 2008); in other words, as what is necessary for successfully teaching mathematics.

It is prudent to assume that teachers and students can share content authority and process authority in a pedagogy in which both students and teachers participate equally in a classroom community. However, as the teacher and students negotiate their practice (see section 2.2.1) to advance their mathematics knowledge, their mutual relations of interdependence would recognise that some aspects of process authority such as mathematics knowledge for teaching will best reside with the teacher. Due to their training and experience in the profession, the teacher is more likely to possess knowledge such as the exam board requirements that influence the questions students practice in class.

2.3.2.2 Positioning

In her book chapter "Doing Undergraduate Mathematics: Questions of Knowledge and Authority", Solomon (2009a) discussed how students are "positioned" in mathematics learning communities by their perceptions of authority. The positioning of students in the classroom can result from how the pedagogy distributes authority between the teacher and students in a mathematics classroom. Davies and Harré (1990) described positioning as the discursive process in which speech and action are used to arrange people in social structures by locating them in conversations as "observably and subjectively coherent participants in jointly produced storylines (discourses)" (p. 48). Storylines (discourses) "are the ongoing repertoires that are already shared culturally or they can be invented as participants interact"

(HerbelEisenmann et al., 2015, p. 188). Interactions are communications, dialogue, or actions that occur among people, either face to face or through other media. Interaction occurs in a mathematics lesson between participants, whether between teacher and student or student and student. As participants interact, they assign positions for themselves and others participating in the interaction.

Positioning constrains what one may meaningfully say or do. With every position comes a connected discourse. In this way, positioning may "diminish the domain of what one does out of the possibilities of what one can do" (Harré & Slocum, 2003, p. 106). There are many positions available for the students and the teacher formed by their interactions in the discourse of schooling. A teacher standing in front of the class positions themselves as *in* authority (process authority) and consequently positions the students as subject to such authority. This positioning of the teacher constrains them to control the students' behaviour, while it expects the students to behave in a certain way, such as sitting quietly. Subsequently handing the whiteboard pen to the students, the teacher is able to position the student as an authority (epistemic authority) in a way determined by the particular context; having been so positioned by the teacher, the student is expected to answer correctly. In this sense, people are positioned through interaction with others, and this positioning tracks these interactions (Davies & Harré, 1990). Positions are responsive to context, and participants' relations to them are dynamic, as one can occupy more than one position and shift between positions.

To position someone is to establish what their duties and rights are, and to determine what they are obliged and allowed or not obliged and not allowed to do (Harré & Moghaddam, 2003; Harré & Slocum, 2003). A participant's rights constitute what others must do for them, and their duties constitute what they must do for others.

Having been positioned, either interactionally by others or reflexively by themselves, a person “sees the world from the vantage point of that position” (Davies & Harré, 1990, p. 6). The position gives meaning to the participants’ and others’ speech, writing, and actions (Harré & Moghaddam, 2003). The meaning of a position is influenced by and influences the past, present, and future of the participants’ interactions and participation; thus, in an educational setting such as a classroom in which the teachers are in authority, the conventional positioning of students can include or exclude them from participating in mathematics learning (Solomon, 2007, 2009c). Positions are defeasible (Harré & Slocum, 2003) and can be disputed over time or in the moment. This study aims to develop a pedagogy that challenges the teacher-student discourse that positions the teacher as knowledgeable and the students as not knowledgeable.

A useful distinction for my thinking is that between position and roles. In contrast to flexible and situation-specific positions, roles in interactions are static, though longterm positions approximate the status of a role (Harré, 2012). The static nature of a role can be understood when considering its close relationship with the function of a “job”. A role, like a job, “represents a set of constraints and requirements that is rather pervasive in someone’s life” (Harre & Slocum, 2003, p. 104). “Teacher” is a fixed role in a school, while the teacher themselves can, through their interactions, be positioned temporarily or lastingly as an authority or otherwise in different situations, dependent on the discourse. This study, by proposing a pedagogy that it takes to be innovative, follows the heels of other research that has tried to change mathematics classroom pedagogy in England by challenging existing authority relations. The pedagogy and its discourses determine the location of authority, as well as the roles and positions available to its subjects in the maths classroom.

2.3.3 Summary

This section examines the pedagogy critiqued by Paulo Freire and Jacques Rancière and its similarities to the conventional pedagogy experienced by students in many present day classrooms in England. The proposed innovative pedagogy will seek to facilitate co-participation and interdependence between students and teachers (Benne, 1970), as against the established forms. Students and teachers sharing authority in the classroom will learn from each other and negotiate how best to use their different skills and experiences in mathematics learning.

In the first three sections of this chapter, I have described aspects of agency relevant to the aims of this study, emphasising the usefulness of the deep constructivist notion of epistemic agency, according to which students take responsibility for their knowledge. I have also discussed the knowledge-creation metaphor, which represents learning as both an individual and collective endeavour; this metaphor prepares the way for the possibility of a dynamic pedagogy, where learning occurs as students interact, rather than where knowledge is merely transmitted into their passive minds by a teacher, as described in section 2.2. Wenger's social learning theory allowed me to examine how learning can occur through students' participation in a mathematics classroom. In this third section, I developed the notion of authority in the context of mathematics pedagogy. In the following two sections, I will begin to argue for the notions and concepts that I rely upon in working to achieve the aims of this study, first constructing the theoretical framework.

2.4 Theoretical Framework

The aim of this study is for the students in my mathematics classroom to actively participate in all aspects of their learning, and to thereby improve their relationship with and their learning of mathematics. To achieve this, existing constructs that have achieved similar aims to mine will be considered in order to help develop the theoretical argument that will underpin this study. The two focal ideas of knowledge building and shared epistemic agency will be introduced in this section as they build upon the previously discussed notions of agency – in particular epistemic agency – that made visible the possibility of students taking responsibility for their learning introduced in section 2.1. The theory of communities of practice reflects this study's interest in the classroom as a learning community and its possibilities for changing participants' relationship with mathematics; however, the community of practice alone cannot account for the acquisition of knowledge, such as mathematics knowledge, and has been supplemented with social learning theories.

2.4.1 Knowledge Building/Knowledge Creation

This section discusses in further depth Scardamalia and Bereiter's conceptualisation of knowledge building and Nonaka's contemporary account of knowledge creation. The concept of knowledge building is helpful for this study as it illuminates students' engagement with knowledge to the extent that it is useful to all classroom participants. It goes beyond the weak constructivist notion of learners' active construction of knowledge to include the two characteristics of intentionality and community knowledge (Scardamalia & Bereiter, 2010) addressed in sections 2.1 and 2.2 respectively. From a weak constructivist perspective, learning is personal and occurs unconsciously through engagement in activity. By contrast, the deep

constructivist perspective of knowledge building considers students as intentionally producing purposeful and valuable knowledge; it furthermore concerns the creation of knowledge in the form of conceptual artefacts for the benefit and advancement of the community. Although individual learning could occur in the process, it is not the ultimate goal of the activity; the primary goal is to solve problems, develop new thoughts and ideas, and advance community knowledge.

Understanding knowledge building requires a prior understanding of conceptual artefacts (Bereiter, 2002, p. 64) and their role in collaborative knowledge building. Conceptual artefacts are abstract knowledge objects (e.g., ideas, theories, algorithms) that can be realised in some material form, typically through discussion or physical construction. Logical relations exist between conceptual artefacts; for example, one conceptual artefact could justify another, and be derived from yet another. Artefacts can be criticised, tested, and improved. Bereiter and Scardamalia claim that in order for conceptual artefacts to be treated as objects of new knowledge and credited as evidence of knowledge, they must: i) be of value to people other than the individual; ii) have value that endures beyond the moment in which it is conceived; iii) apply beyond the situation that gave rise to them; and iv) display evidence of a modicum of creativity in their production (Bereiter & Scardamalia, 2011, p. 3). For example, consider a situation in which an individual, through experience as a decorator, develops a good sense of symmetry. For Bereiter and Scardamalia, the individual has *acquired* knowledge, not built it. If the individual produces a short video that shows how images are reflected from one side to another, the individual would be said to have *created an artefact*. This artefact, though not conceptual, would enable others to access and acquire the tacit knowledge and skills that the individual has. For the artefact to be termed

conceptual, the individual would have to produce a mental *theory* that explains how the symmetric image is produced. This theory is a conceptual artefact, and it can be treated as knowledge that is represented in the video, which therefore fulfils the criteria above. Developing the theory that supports the conceptual artefact is the process of knowledge building. When students build knowledge, they are actively engaged, as a community, to create conceptual artefacts. This collective approach to creation shares and advances the knowledge of the community.

Knowledge building therefore consists in the continuous collective production of improved forms of ideas (conceptual artefacts) that contribute to the advancement of knowledge in a community (Bereiter, 2002). It challenges learners to go beyond individual capabilities and to collaborate, with whom they share a common epistemic goal. Bereiter (2002) and Scardamalia & Bereiter (2014) derived knowledge building from an epistemological outlook that treats ideas as entities in their own right, independent of the mental states of individuals. In classrooms organised around knowledge-building pedagogy, individual students are recognised for their contributions to collective knowledge advancement rather than for what is “in their minds”. In these classrooms, students find respect and acceptance as contributors in knowledge creation (Scardamalia & Bereiter, 2006).

Thus, on the basis of their theory of knowledge building, Scardamalia and Bereiter proposed a pedagogy that encourages an individual to intentionally execute higher level cognitive processes on their own, without depending on their teacher, within a classroom community that further sustains knowledge advancement by providing opportunities for student-to-student feedback. The pedagogy is based on twelve principles (see Appendix 1) which deviate from currently prescribed procedures (Lai

& Campbell, 2018; Scardamalia, 2002). Six of these principles align with the aims of this study and the innovative pedagogy I propose. The other are less relevant to secondary school mathematics pedagogy that follows the GCSE curriculum. I will here outline the principles that align with this study, and subsequently synthesise them with other active theories in order to produce my own characterisation of shared epistemic agency in a knowledge-building pedagogy. The relevant principles are:

- *Community knowledge, collective responsibility* that encapsulate the aim of knowledge-building pedagogy to produce knowledge that is useful to and usable by the participants of a classroom community (see section 2.2.2 on communities of practice).
- *Epistemic agency* (see section 2.1.3), which is essential for supporting the collective efforts of knowledge advancement beyond the individual performance of tasks.
- The *collective improvement of ideas* (see section 2.3.2.1). There are no final truths; learners view every idea as having the potential to be improved. The improvement of ideas comes from the students as they seek to reconcile conflicting conceptions. There is the "continual application of a 'make it better' heuristic" (Scardamalia & Bereiter, 2014, p. 400).
- *Knowledge-building discourses* for the improvement of ideas (see section 2.2.1). Bereiter (2002) argues that classroom discourse should mimic professional science discourse. It should, in other words, be cooperative and more concerned with creatively advancing the collective knowledge beyond what is currently known.

- The *democratising of knowledge* that is a result of such discourses (see section 2.3.2.1). In a classroom based on the knowledge-building paradigm, all participants are deemed legitimate contributors to collective knowledge.
- The *use of authoritative information*, such as multimedia resources, in these classrooms. In my classroom, this involves the careful use of such things as MathsWatch, textbooks, and other media in order to construct coherent knowledge from diverse representations.

The following section will discuss the concept of "knowledge creation", not to be confused with the "knowledge-creation" metaphor for learning described in section 2.2.1.

2.4.1.1 Knowledge Creation

Though distinct from knowledge building, Nonaka's (1991) concept of knowledge creation relates to the former in its focus on the ways in which a community can create new knowledge from within, through active engagement; this concept is useful for the secondary mathematics classroom in which students need to make mathematics knowledge and their problem-solving strategies explicit to each other. The distinction between knowledge building and knowledge creation is due to the different disciplinary commitments of the associated theorists: knowledge building was developed in the context of education, while knowledge creation is a dynamic that was initially identified in the context of the corporate organisation.

Nonaka's concept of knowledge creation is germane to the aims of this study to the extent that it recognises the value of knowledge as both explicit and tacit, placing an

emphasis on the process by which personal knowledge is made available to others. Explicit knowledge is easy to articulate, while tacit knowledge is personal, hard to formalise, and challenging to communicate to others; it consists of mental models, beliefs, and perspectives (Nonaka, 1991). This concept can explain how, in the mathematics classroom, knowledge can be tacit or procedural, and students may find it difficult to articulate their reasoning and justify their solutions to problems; or else the knowledge can be explicit, in which case students will typically find it easy to communicate their thinking. Both types of knowledge are of value, and Nonaka's theory further reveals the process by which the two interact in a "spiral of knowledge" (p. 97) to generate innovations; that is, to create new knowledge. This presents the interaction between students as a process of knowledge creation

The knowledge spiral, which depicts the iterative transformation and sharing of knowledge from the level of the individual to that of the organisation, and even among organisations, is grounded in four complementary knowledge-creation stages that operate between individuals and groups in an organisation (Figure 2.5).

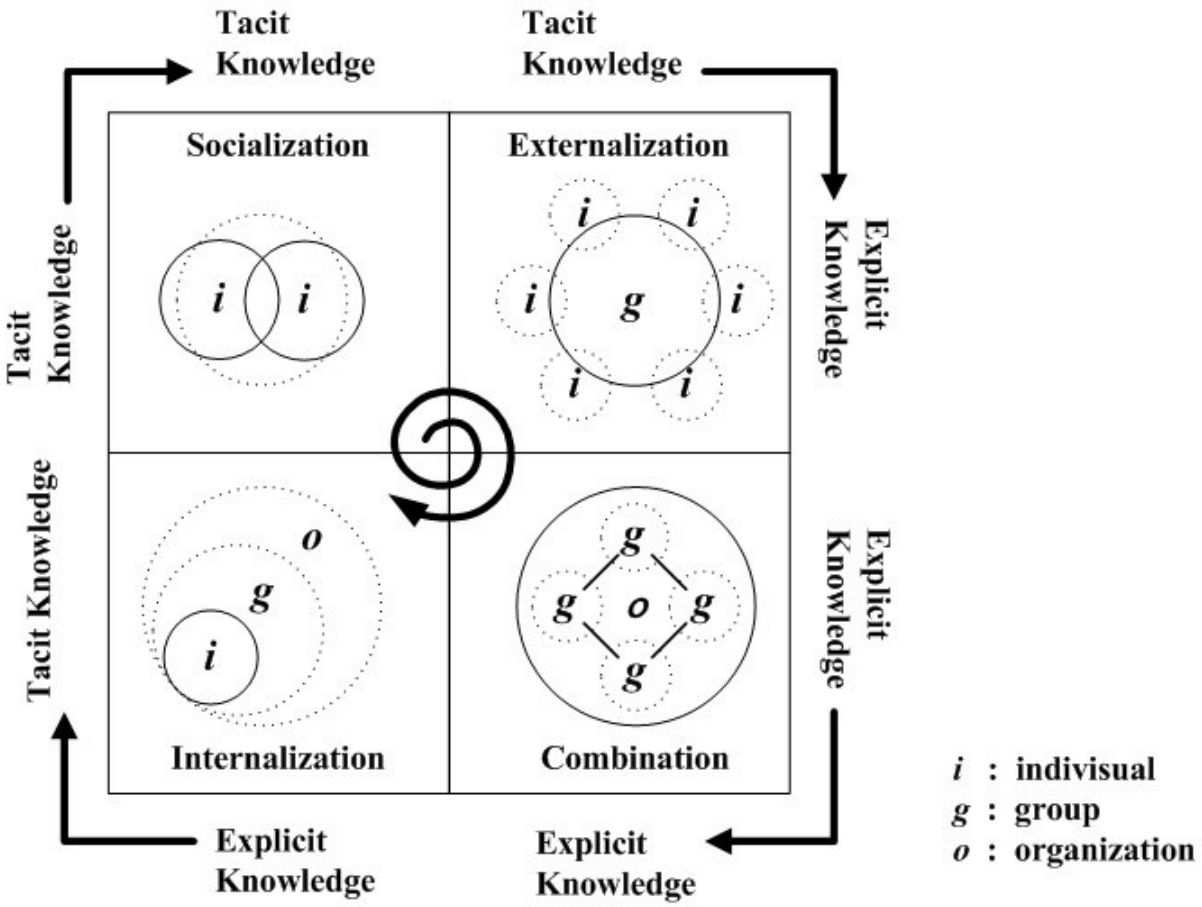

Figure 2.5 – The Knowledge Creation Spiral. Source: (Umemoto, 2002, p. 464)

The first stage involves the transmission of tacit knowledge from individual to group due to the sharing of experiences in the activity *socialisation*. It is essential to develop trust between individuals at this stage, as close interaction and collaboration are necessary for the effective sharing of the explicit knowledge over time. In the second stage, tacit knowledge is transformed into explicit knowledge through *externalisation*. In this stage, the tacit knowledge of a socialised group is made explicit through discourses, metaphors, diagrams, and concepts – that is, through artefacts. Thus, during externalisation, knowledge can be exchanged by means of what Nonaka refers to as a "metaphors, analogies and models" (p. 99), which is

broadly analogous to Bereiter's conceptual artefact. In the third stage, the new explicit knowledge aggrandises itself through its *combination* with existing explicit knowledge, and is subsequently distributed throughout the organisation. In the fourth stage, explicit knowledge is transformed back into tacit knowledge, through *internalisation*, and begins to inform the practices of individuals. This implicit knowledge is then itself socialised, beginning the cycle anew.

Bereiter (2002) was critical of Nonaka's knowledge spiral on four counts, noting its exclusion of creativity, understanding, knowledge work, and collaborative knowledge building (Bereiter, 2002, pp. 175–177). He argued that as the model does not distinguish between "knowledge involved in productive work and knowledge that is a product of productive work" (Bereiter, 2002, pp. 177–178), it cannot promote learning that will contribute to a community's ability to create knowledge. He noted that the knowledge spiral could be a carrier of ritual and tradition, as it presupposes shared implicit understanding but does not necessitate understanding at the individual level. The individual does not become what he referred to as "a fully functioning member of a knowledge society" (Bereiter, 2002, p. 173). However, I argue that Nonaka's perspective on knowledge can contribute its thinking to structure the mathematics pedagogy that I seek to develop in this study. Supplementing her picture of the transformation of knowledge with capacities for discussion and shared problem solving evades Bereiter's critiques and contributes individuals' tacit knowledge to community knowledge. In this sense, student-to-student explication of mathematical knowledge fulfils the criteria of new knowledge, and I argue that it qualifies as knowledge building. Despite Bereiter's criticism, other authors such as Paavola et al., (2006), whom I discussed in section 2.2, and Damşa et al., (2010) whose work I describe in the next section, have also combined these two models.

In summary, knowledge building and knowledge creation orient the design of a pedagogy that focuses on the individual's engagement with knowledge for community benefit. Individuals can be seen to benefit from the pool of knowledge within the community from which they can draw. This picture of the synergy of the individual and the community is in agreement with Wenger's theory of community of practice (see section 2.2.2), wherein he argues that through participation, benefits such as accountability and mutual relations contribute to the advancement of a community's enterprise (Farnsworth et al., 2016). In the following section I describe the kind of agency I desire the pedagogy of this study to develop in the students. This agency is referred to as shared epistemic agency.

2.4.2 Shared Epistemic Agency

Shared epistemic agency, introduced by Damşa et al. (2010), is the central concept of this study. It is described by these authors as an emergent construct that builds on Scardamalia's (2002) notion of epistemic agency (see section 2.1.3), which they used to characterise undergraduate students' abilities to carry out complex, authentic collaborative projects. They conceptualised shared epistemic agency to include the notion of sharedness that presupposes intentionality (Bandura, 2001; see section 2.1.1), the collaboration between participants, the social-communicative processes that leads to new collective knowledge (see section 2.4.1), as well as the notion of an established community of practice – i.e. the mutual relations of participation that support coherence in a community (Wenger, 1998, and section 2.2.2). Shared epistemic agency describes the interdependency of partners (see section 2.3.2.1) and the collaborative actions that do not happen when individuals work on their own. It also draws on the knowledge-creation perspective of learning (see section 2.2.1)

that situates learning as occurring during collaborative practices that create shared material knowledge objects.

Damşa et al.'s construct of shared epistemic agency, which lies within the knowledge-creation perspective (see section 2.2.1), depicts a specific form of epistemic agency (see section 2.1.3) that emerges during collaboration to create shared knowledge objects. In this sense, the shared knowledge object is both the outcome of the group's collaboration and the reason for the group's activity (Stahl, 2009, p. 64). Damşa et al., like Nonaka (1991), acknowledge the interaction between explicit and implicit knowledge as of value to knowledge creation, while arguing that shared epistemic agency goes beyond knowledge building. They argued that knowledge building emphasises collective collaboration for the improvement of singular ideas, whereas shared epistemic agency involves working on more than one idea to create knowledge through the advancement and development of complex knowledge objects (Damşa et al., 2010). These authors posit that learning occurs as students act to give conceptual artefacts a concrete form as material objects of shared knowledge, such as reports, essays, or software.

Shared epistemic agency can be understood as the "capacity that enables individuals, groups, or collectives to make appropriate judgments, to make plans, and to pursue these through purposeful action, in order to achieve the construction of knowledge" (Damşa, 2014, p. 446). In addition to sharedness, this definition emphasises epistemic productivity and negotiation within the community. The related notion of "temporality" refers to the emergent nature of the agency in question (p. 447); it suggests a certain kind of practice that is reflexive and iterative, considering past practices and experiences metacognitively to solve present problems and create plans that lead to future desired outcomes.

Shared epistemic agency is an empirical concept; in other words, it is a conceptualisation of observable phenomena and they expressed the intentions that materialise indicative of the agentic behaviour (Damsa et al., 2010, p. 155). The unit of analysis, is, therefore, the group-level actions that constitute the conditions for its emergence. These actions fall into two categories: the epistemic and the regulative. Epistemic actions are directed towards knowledge and the creation of knowledge objects. These include actions that serve to create awareness of the current knowledge situation within the group (e.g., brainstorming, discussing); that create shared understanding; that alleviate a lack of knowledge and gather information (e.g., researching, asking, discussing); and that generate collaborative actions (e.g., explanations, concepts) (Damşa & Andriessen, 2012).

Regulative actions are the processes that occur at the metacognitive level and that prepare the foundation for epistemic actions. They do not directly influence the creation of knowledge objects, although they make their creation possible. Regulative actions are based on the group's intentions (Bandura, 2001) to create the knowledge object, and consist in the procedures that occur as a result of this intention (Emirbayer & Mische, 1998); that is, they are the result of the metaknowledge that the group has about the process and the progress of creating the knowledge object that informs the actions that the group takes. These actions, consisting of projective actions, the setting of a common goal, the creation of a plan of action, and proactive engagement, are required for successful outcomes. Regulative actions, such as monitoring the progress of the knowledge object and reflecting on it, and relational actions – the social aspect, i.e., validation and the acknowledgment of individual contributions – facilitate relations between individuals

and the group, making possible the maintenance of their epistemic community. An overview of epistemic and regulative actions is offered in Appendix 5.

2.4.3 Summary

Knowledge building conceptualises a community learning environment in which students interact with shared intentions to improve on their ideas, creating new knowledge continuously. Shared epistemic agency is a conceptualisation of the capacity of individuals and collectives to perform collaborative actions, bringing together multiple ideas to create a knowledge object, which is the material realisation of their new knowledge. To achieve the aims of this study, I consider these concepts in the context of the capabilities of students. It is my intention to promote the emergence of shared epistemic agency amongst the students in my mathematics classroom, creating a learning environment in which they continuously develop new knowledge and control their own knowledge advancement.

Although Damşa et al. describe shared epistemic agency in terms of the epistemic and regulative actions that, over time, lead to the creation of a knowledge object, their empirical study reports only on undergraduate students engaged in one-off collaborative group work to produce an authentic knowledge object such as an instructional design project or a training and evaluation project (Damşa et al., 2010). Their research cannot be applied without modification to a secondary mathematics classroom, in which both participants and subject matter are considerably different from the original objects of the study. Thus, I proceed with my own study by apprehending and developing the notion of shared epistemic agency in this new context; I determine that the shared epistemic agency that I want to emerge is a quality of students that is an index of active participation in all aspects of their

learning of mathematics and an improved relationship with mathematics, which leads to improved mathematics learning. Good GCSE grades will evidence this improved learning in the students' terminal secondary school examinations.

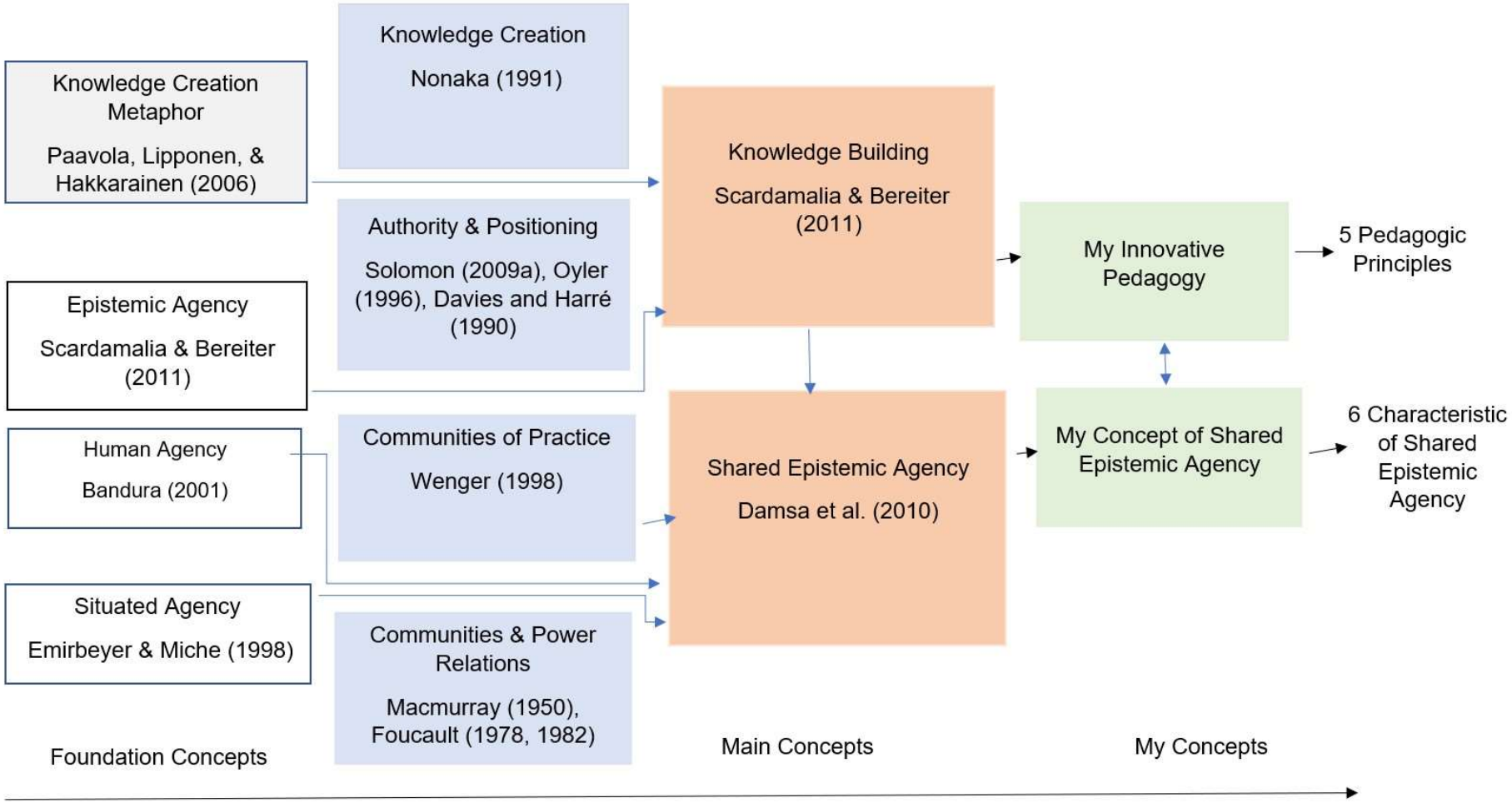

Figure 2.6 – Theoretical background: interconnection and relevance of concepts, notions, and perspectives.

On the strength of the theoretical background developed) in this chapter (see Figure 2.6), I can now characterise the specific kind of shared epistemic agency that I consider appropriate for the aims of this study. Its six characteristics are given as:

a) *Intention*. The agency will include intentionality: the proactive commitment to bring about a desired outcome (see section 2.1.1) that presupposes purposefulness and will include community knowledge (cf. Bandura, 2001;
Damşa et al., 2010; Scardamalia & Bereiter, 2014).
b) *Extension*. The student deliberately focuses on going beyond existing knowledge. This notion originates in the theory of knowledge building (see section 2.4.1, first paragraph) that extends constructivism towards deep constructivism (see section 2.1.3), in line with which students control all aspects of learning (cf. Bereiter & Scardamalia, 2011).
c) *Explication*. This refers to purposeful dialogue that makes knowledge explicit so that it can be shared (see section 2.4.1.1). Drawing on Nonaka's knowledge spiral, shared epistemic agency will acknowledge sharing personal knowledge and the interaction between tacit and explicit knowledge that communicates mathematics knowledge through dialogue, advancing all students' knowledge in the classroom (cf. Nonaka, 1991).
d) *Expertise*. Students are considered to be expert learners who set themselves similar tasks to those typically imposed by mathematics teachers. This draws on Damşa et al.'s notion of regulative actions

(see section 2.4.2) that depict the metaknowledge possessed by the group that allows them to manage and monitor the advancement of the knowledge object, requiring them to not to

rely solely on external sources such as the teacher (cf. Damşa & Andriessen, 2012).

e) *Mutual Relations*. In order to sustain epistemic agency, mutual relations between individuals must be established (see section 2.2.2). The application of my revised notion of shared epistemic agency will include a consideration of the mutual relations that support the coherence of the community in the project of fulfilling their common purpose of learning mathematics (cf. Wenger, 1998).

f) New Knowledge – This refers to learning through collectively developing ideas and explanations that are new to the students (see section 2.4.1) The final object of analysis will be the new knowledge students are able to create, in the form of a conceptual artefact that is the product of more than dialogue with the pedagogical authority, instead combining the collective and individual contributions of learners who are actively engaged in developing new ideas and explanations in the context of unfamiliar mathematical concepts (cf. Bereiter, 2002; Bereiter & Scardamalia, 2011).

The precise nature of these characteristics, in the specific context of the knowledgecreating classroom practices that are the object of my study, will be illuminated in the following sections. The actions and artefacts that are indicative of each of these six characteristics will also be identified by the end

of this study. Henceforth, the term “shared epistemic agency” will encapsulate the six characteristics stated above. The wider construct originating in Damşa et al. (2010) will be referred to as “SEA” for differentiation. Therefore, a preliminary question that this study seeks to answer is:

What are the indicators of shared epistemic agency in the mathematics classroom?

As previously stated, knowledge building requires a learning environment that could support the emergence of shared epistemic agency. The innovative pedagogy I propose draws on the concepts of knowledge building and knowledge creation to support the emergence of shared epistemic agency. The pedagogy will be based on the knowledge-creation metaphor of learning, according to which new knowledge is continuously and creatively produced from within the learning community. It will seek to reimagine the conventional teacher-student power relations by demonstrating the interdependence of authority (see section 2.3.2.1), and by redefining learning as a community endeavour. The pedagogy will draw on the six key principles of knowledge building, and will notably include reflection that leads to improvement (see section 2.5.1), as well as explicitly relying on the community relations that support the genuine advancement of knowledge. Given my synthesis of the previous literature performed in this chapter, I clarify the principles of the innovative pedagogy I propose as stipulating that students are responsible for:

1. Building objects of mathematical knowledge (cf. Bereiter, 2002; Damşa et al., 2010; Emirbayer & Mische, 1998; Reed, 2001; Scardamalia, 2002).

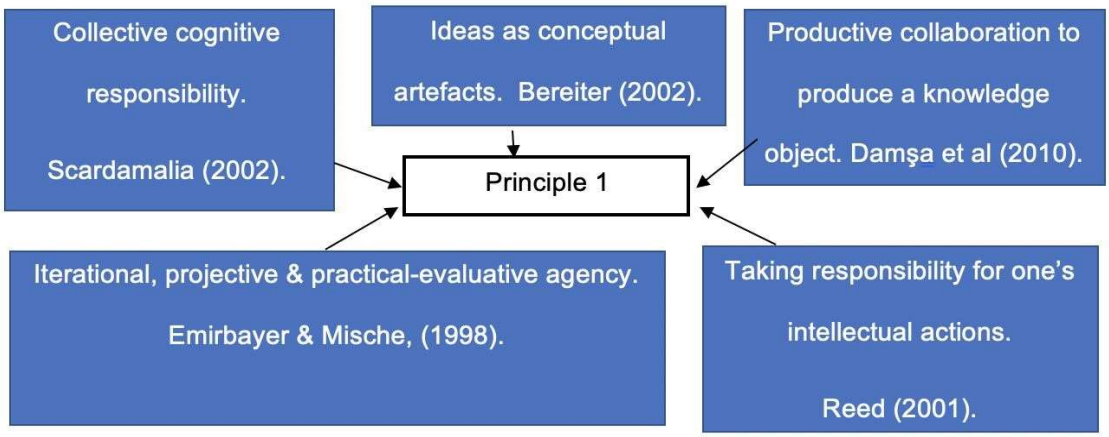

Figure 2.7 – Pedagogic principle 1

2. The process that makes this knowledge explicit so that it can be shared, internalised, and used by all the classroom participants (cf. Bandura, 2001; Damşa et al., 2010; Nonaka, 1991).

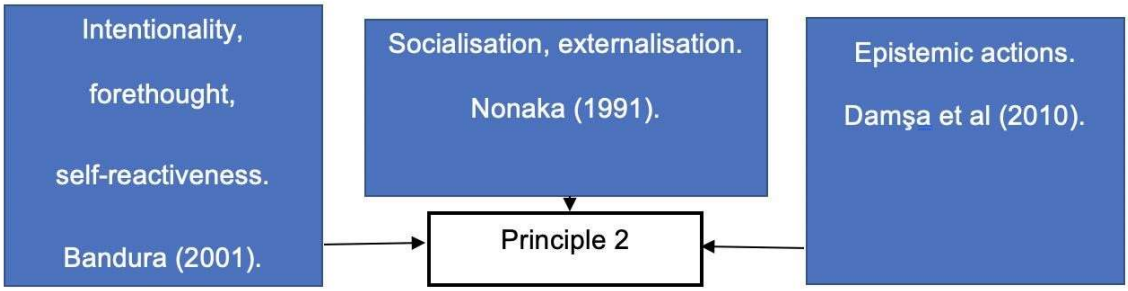

Figure 2.8 – Pedagogic principle 2

3. The discursive process that communicates this knowledge to the classroom community (cf. Emirbayer & Mische, 1998; Nonaka, 1991; Scardamalia & Bereiter, 2014).

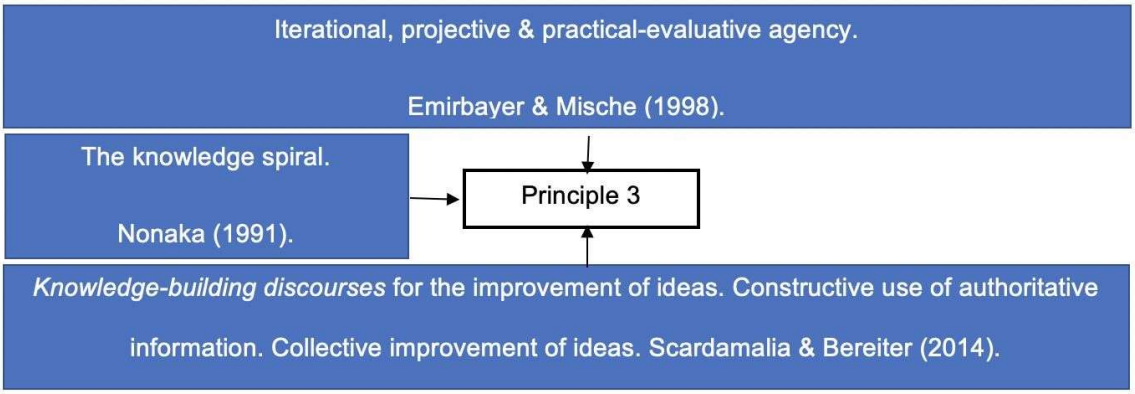

Figure 2.9 – Pedagogic principle 3

4. Maintaining the social relations and communicative processes that are conducive to the advancement of mathematical knowledge (cf. Bandura,

   2001; Damşa, 2014; Damşa et al., 2010; Wenger, 1998).

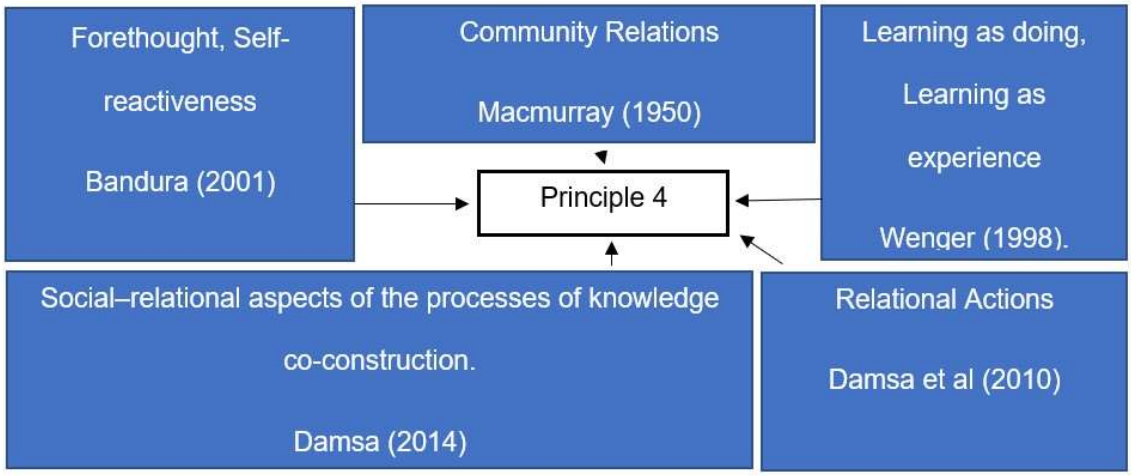

Figure 2.10 – Pedagogic principle 4

5. Reflecting on practice and making plans for the improvement of ideas and activities (cf. Bandura, 2001; Bereiter & Scardamalia, 1998; Emirbayer & Mische, 1998; Yang, Chen, et al., 2020).

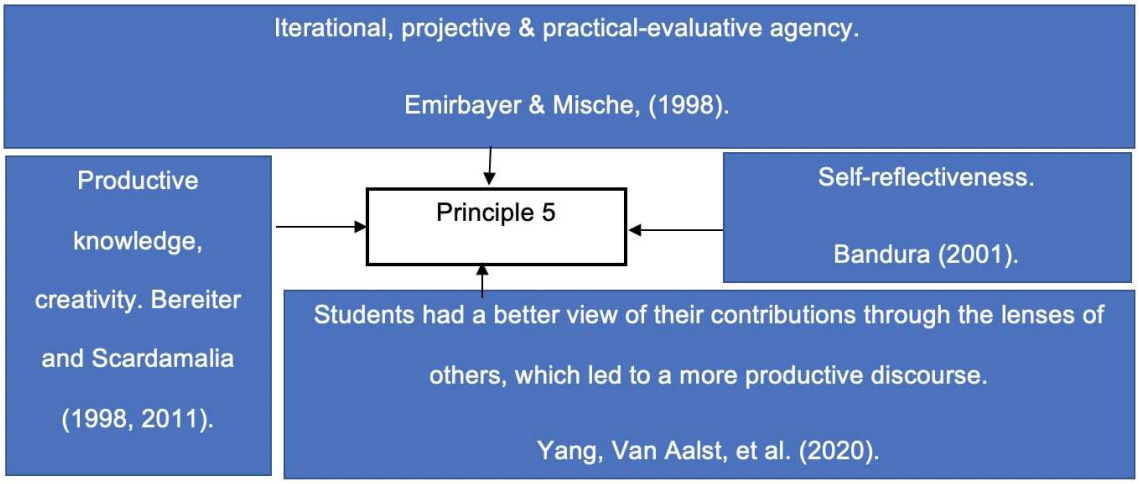

Figure 2.11 – Pedagogic principle 5

In the next section, I will investigate pedagogies that have turned control of learning over to the students, providing a touchstone for my own suggestion of a pedagogy that meets the aims of this study in the context of my mathematics classroom.

2.5 Researching Innovative Pedagogies

The emergence of shared epistemic agency requires more than groups of individuals learning collaboratively. Simply bringing students together to work on a joint task and pooling their knowledge together is not sufficient to create new knowledge (Barron, 2000; Scardamalia & Bereiter, 2010). As elaborated in the previous section, it requires an established community with customary practices negotiated over time (Damşa et al., 2010; Wenger, 1998). It necessitates an innovative pedagogy with a purpose, namely, which goes beyond collaborative learning to include the notion of productivity; that is, a knowledge-creating classroom. Having outlined the principles of my innovative pedagogy, in this section I

investigate knowledge-building pedagogies and transformative mathematics pedagogies in England to inform the design of my own.

2.5.1 Knowledge-Building Pedagogies

This section describes three pedagogies (Moss & Beatty, 2011; Yang, Chen, et al., 2020; Yang, van Aalst, et al., 2020; Zhang et al., 2018) that are explicitly framed by the concept of knowledge building that I described in section 2.4.1. Though online technology, which is not a focus of my own study, heavily supports student interaction in these pedagogies, the findings are still relevant for their analyses of the ways in which the pedagogy was decisive in developing students' participation in the creation of new knowledge. In Moss and Beatty (2011), fourth-grade students collaborated on an online database that provided a communal space where students posted their ideas and read each other's, engaging in critical reflective activity. In this way, they all contributed to the community knowledge base. The database was entirely student-managed; the teacher's voice was not present, nor were answers or solutions provided from an external source. In other words, the students had collective cognitive responsibility for coming up with conjectures, and solutions, and negotiating the various approaches to mathematical problem solving.

This research illustrates how the knowledge-building principles of the democratisation of knowledge and epistemic agency (see section 2.4.1) can further a mathematics problem-solving and learning culture. Democratising knowledge requires that all participants within a community

are legitimate contributors to the community knowledge and that their contributions are valued and acknowledged. Moss and Beatty's students working together to solve problems evidence their epistemic agency; they supported each other's suppositions and questioned when ideas or solutions were incorrect. In this way, the community was assured that the solutions provided were correct. In the absence of an external verifier, the students not only verified their solutions to the problems independently, but also routinely took responsibility for offering evidence and justification for their solutions, with the intention being to make sure that the whole community understood the proper solution to the problems. In this way, they took responsibility for the community's collective understanding. Moss and Beatty researched 8-to-9-year-olds across three schools; the intervention took the form of a one-off addition to the existing classroom pedagogy, which contrasts with my aim to change the overall learning experience of secondary students for a single subject over a whole year.

Moss and Beatty's research, however, does bear similarities to my study; its demographics were of a similar economic status, and a significant proportion of the students were categorised as low-achieving. Equally, the democratisation of knowledge and the quieting of the teacher's voice are outcomes of a knowledgebuilding pedagogy that resonate with this study's aims.

Yang, van Aalst, et al., (2020) conducted research with low-achieving ninth-graders who collaborated on an online platform. The findings from this research were similar to those of Moss and Beatty. They illustrate how "academically low-achieving" students could get involved in sustained

collaborative and productive knowledgebuilding discourse and inquiry (p. 1253). In addition, in a manner that is particularly relevant to my study, the research illustrated that by engaging in reflections, students had a better view of their contributions through the lenses of others, which led to a more productive discourse. Reflecting on others' contributions to knowledge improvement did not lead to criticism, but became the community practice, the classroom norm. This research focused on developing a community in which the goal and focus of the classroom was knowledge-building collaboration that advanced collective knowledge; reflective assessment was not based on individual attainment, but on the progress made by the whole class.

Research by Zhang et al. (2018) sought to support student-driven inquiry within a socially organised pedagogy. The researchers worked with two upper-primary school classrooms on a knowledge-building initiative. The researchers sought to provide structure to the students' inquiries while still allowing the flexibility that enabled their agency and imagination to thrive. The researchers designed an inquiry-structuring, timeline-based web platform, ITM (Zhang et al., 2018, p. 401), that discovered emerging directions and interests in students' interactive discourses. ITM then formulated unfolding inquiry strands and made them visible to students to support ongoing participation and reflection. The reflective process, facilitated by the technological apparatus, shifted control of the inquiry from the teacher to the student's agency. While the research highlighted the value of reflection, and knowledge building pedagogy was the established science pedagogy for a twelveweek period, the teacher guided the students' inquiry to

a larger extent than is proposed in this research. The research shed light on how to construct pedagogical structures with students to develop a classroom community that sustains the students' ownership of their collective thinking journey to support knowledge-building interaction, but I attempt to go further, in line with the renunciation of authority consistent with deep constructivism.

The three studies noted above show how a knowledge-building pedagogy can lead to the emergence of favourable characteristics in the classroom environment, such as the democratisation of knowledge, epistemic agency, the quietening of the teacher's voice, community learning, and improved participation in learning – however, in each case, a technology platform where ideas were shared was central to the pedagogy. In addition, the three studies took place outside of England. In the following section, I will therefore conduct a literature review to identify further research related to my study that has transformed pedagogies, without reliance on a technology platform, in English secondary schools.

2.5.2 Transformative Pedagogies in England

My literature review focuses on studies that have transformed mathematics pedagogies in secondary schools in England in the last ten years, as this frame bears close relevance to the context of this study (see section 1.1). I used the UCL library search facility and put in the terms: <*Any field (contains)* ***transformative pedagogies*** *AND Any field is (exact)* ***mathematics*** *AND Any field is (exact)* ***England***>**,** and I filtered for the Years: 2011-2021, Form: *Articles and Book Chapters*, and Topic: including Pedagogy. Two articles from the 145

results were of interest; the other 143 did not describe a mathematics pedagogy in England. However, on further reading, these two were not found to be germane to the specific aims of my study.

Ruthven et al. (2017) developed the "epiSTEMe" pedagogical model, which focused on improving student engagement with mathematics and science in the first year of secondary school education through exploratory dialogic conference. It was not relevant to this study, as the pedagogic measures it proposed retained the privileged position of the teacher as an authority, and it involved changing the nature of mathematics content as opposed to improving student agency. This research, if anything, further entrenches the roles of teachers as knowledgeable and students as requiring continuous guidance to be knowledgeable.

The "participatory pedagogy" of Lyndon et al. (2019) focused on pedagogic mediation and viewed the student as a social being with the capacity to construct their knowledge in collaboration with others. Despite the similar view of the student in my study, the research differed in context as it focuses on nursery school children, and was not mathematics-specific.

I altered the search term to: <*Any field is (exact)* ***pedagogy*** *AND Any field is (exact)* ***mathematics classroom*** *AND Any field is (exact)* ***England*****>.** I filtered for the Years: 2011-2021, Form: *Articles and Book Chapters*, and Topic: including Pedagogy, including Education & Educational Research. This produced 13 results; of interest was the work of Hofmann & Ruthven (2018); Watson

& De Geest (2014); and Wright et al. (2020). The other 10 articles did not describe a mathematics classroom pedagogy in England.

Watson & De Geest (2014) carried out three-year ethnographic research with three secondary school mathematics departments in England, teaching students of a similar socioeconomic background to that of the students in my study. The departments sought to improve the achievement of their students. However, the transformation did not directly focus on improving student agency. Instead, it centred on changing classroom groupings to mixed-ability, expanding the mathematics tasks available to students, and developing teachers' confidence in their subject content knowledge. These changes are similar to those that have been discussed in my mathematics department and many others over the years; with this study, I propose something more radical: a change in our beliefs about students and the historicocultural role assigned to them.

Hofmann & Ruthven (2018) were co-researchers on the epiSTEMe project, alongside Ruthven et al. (2017); indeed, the limitations of the project noted above apply to their study as well. I discuss Wright et al. (2020) below.

Altering the search term to <*Any field is (exact)* ***mathematics pedagogy*** *AND Any field is (exact)* ***student agency*** *AND Any field is (exact)* ***England***> produced one new result: Wright (2017)***.***

Further manipulation of the search terms revealed Foster (2013), who is critical of the reductionist approach to traditional mathematics pedagogy, and who and calls for a more holistic approach to mathematics tasks;

however, his article was focused on critique, and did not put forward a pedagogy. My systematic search, therefore, resulted in the identification of two studies that share an interest in putting forward a pedagogy, based in an English secondary school, and focusing solely on mathematics. These are the works of Wright (2017) and Wright et al. (2020) from my literature review; the work of Solomon et al. (2021), which I discovered through a search of recent articles from researchers in my bibliography, was also useful.

Wright et al. (2020) adopted a critical model of participatory action research to transform mathematics classroom practice in a London secondary school. The mathematics pedagogy research project they undertook was a collaboration between Peter Wright, an academic researcher, and two secondary school mathematics teachers, who are also co-authors. The project's aims were twofold. The first aim was to investigate the effect of making a progressive mathematics pedagogy visible to students, leading to their appreciation of how to be successful mathematics learners. Progressive pedagogy in this research referred to a problem-solving teaching approach that was discursive, collaborative, and open-ended. The second aim focused on developing and refining the model. Wright's approach to pedagogical transformation focused on developing the teacher's practice.

Wright comes from the school of critical mathematics education, also influenced by

Paolo Freire, whom I mentioned in section 2.3.1. Critical educators such as Gutstein (2006) introduced practices that reimagine the authority

relations in the classroom and alter the mathematics teaching materials in a bid to help students to understand the society in which they live, and recognise how inequality is contested and produced in society. I do not advance a critical view of society; nor am I interested, in this study, in precipitating changes in the social at large. Though my study focuses on social justice in terms of wanting the students to be total participants in their learning, its ultimate aim is improving exam performance to offer students greater opportunities in life. Wright et al. indeed seek a reversal of historically inequitable academic outcomes by making the pedagogy more visible; in this way, their study and mine have a similar focus. However, though he argued for teachers and students to reflect on the implicit power relations in the classroom that prevent a relationship of trust, which would allow classroom rules to be negotiated and made clear to students, rather than the teacher relying on their authority to control students (Wright, 2017), Wright et al.'s transformation did not go far enough in my view. The researchers restricted student's agency to articulating the justification behind the teacher's intentions. The students did not participate in any decision-making, nor did they initiate or direct any change within the pedagogy; this leads me to question whether the intentions to involve students in negotiating classroom rules held the same social learning focus of developing a practice (see learning by doing in section 2.1), as my study intends to do. The locus of the participatory action-research practice was the relationship between the researcher and the teachers.

In Solomon et al. (2021), the research focused on introducing “Realistic Mathematics Education” (RME) to a group of low-attaining students who had not achieved the accepted pass grade in GSCE Mathematics. The development of the RME pedagogy is supported by “guided reintervention” that requires increased participation on the part of the students and particular practices by the teacher, both underpinned by a significant shift in responsibility and authority from the teacher to the students. The teacher orchestrated whole-class mathematical discussions for a specific goal (p. 175-6). The pedagogy positioned the students as knowledgeable and expected them to articulate and defend their solution strategies.

The research shares similarities with this study. It sought to increase students’ epistemic authority by shifting authority from the teacher to the students and positioning them as knowers responsible for articulating their thinking and solution strategies. However, the study was founded upon a curriculum-focused RME theoretical base, whereas my study is driven by pupil relationships with mathematics. I left the question of how the mathematics was to happen to the students, and our own resources built on workbooks and exam practice.

The literature review has shown that numerous researchers in mathematics education have sought and still seek changes to the conventional mathematics pedagogy. Both Wright and Solomon needed longitudinal studies to embed and research their pedagogy, and both were participatory in that they trialled new ideas in existing cultural settings, not labs. My study, however, stands alone in seeking an everyday pedagogy

in which students take control of learning the mathematics curriculum in a secondary school mathematics classroom in England.

The need to change my classroom pedagogy started long before the commencement of this doctoral study. As described in the introduction, I had begun to consider how my actions in the classroom may constrain the students from engaging with mathematics logically. Prior to embarking on this research, I had started to allow the students to take greater control in the classroom and to teach topics to each other. I also allowed them to make decisions about the sequence of the teaching of topics. However, I knew that convincing other professionals to change the conventional pedagogy required a systematic study. I also needed to justify to myself the benefits of my pedagogy by rigorously collecting evidence.

I am aware that there must be other ways of designing a pedagogy that would lead to the emergence of shared epistemic agency in a mathematics classroom. This study's innovative pedagogy started to develop as my classroom practice for two years before the commencement of this study, when I had attempted to silence my authoritative voice as teacher in the classroom so that students could find their own ways of making sense of mathematics through their active participation. In this way, I believed they would respond more logically to problem-solving and ultimately do better in the GCSE terminal examinations.

2.5.3 Summary

My pedagogy will involve the students working collaboratively in line with the pedagogic principles I have established above (see section 2.4.3). The design of the pedagogy will be described in fuller detail in the following section. From my experience before this study, I found that the students act as both an epistemic support and motivator for each other's mathematics knowledge when the authority of the teacher is weakened. The kind of participation that I want my students to be engaged in will develop and change the teacher-student relationship over time. This directs this study towards an action-research methodology that seeks to answer the following questions:

1. What are the indicators of shared epistemic agency in the mathematics classroom?
2. What sustains the emergence of shared epistemic agency in the mathematics classroom?