1 INTRODUCTION

Socio-cultural theories of learning (e.g. Boaler & Greeno, 2000; Lave & Wenger, 1991; Rogoff et al., 1998; Wenger, 1998) suggest that the way in which mathematics learning occurs – that is – the specific practices involved in the learning process, shape and define the knowledge produced, as well as the different relationships students have with this knowledge and the uses they make of it.

Boaler's research suggests that students who participate in negotiating and interrogating mathematics as they learn it are more able to use apply its principles in situations that require such practices when compared with students who learn mathematics by working through exercises from a textbook (2002a). Boaler argues that students' knowledge is applied in situations outside the classroom in a way that is dependent on the situation within the classroom, given that knowledge is coconstituted by how the learning occurred. My study, in this spirit, starts from the position that improving the mathematics knowledge of my students and their relationship with the subject requires a change in their current learning practices.

For the first fifteen years of my teaching career, my classroom pedagogy was similar to that of the mathematic teachers I had encountered, including those with whom I taught. Summarily, my role in the classroom, as the teacher, was to transfer my mathematics knowledge to the students; the students' role was to listen to me and internalise this information; this was the core of the learning process. At the start of the lesson, I would explain a topic to the students and work out a select number of examples on the board; the students then had the opportunity to ask questions.

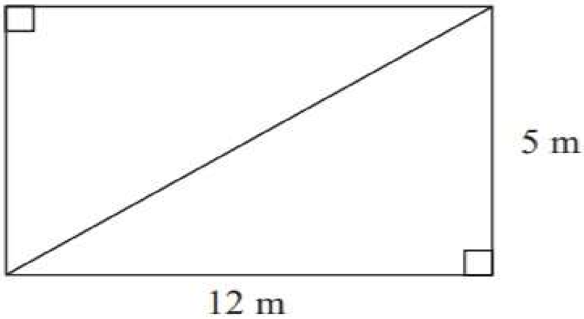
Following questions, the students would try out similar problems relevant to the topic.

At this point, I would supervise the classroom, checking students' work and answering questions that may arise; finally, a summary of our learning (or my teaching) occurred at the end of the lesson.

Due to my years of experience as a mathematics teacher, I became responsible for teaching the students who appeared to have a particular difficulty with learning mathematics. Over time I came to believe that my classroom pedagogy, combined with the nature of the students' apparent difficulties, contributed to reinforcing in the students' minds the conviction that they were not good at mathematics. The anecdotal evidence that formed the basis of my belief was what I viewed as the students' increasing reliance on my mathematics knowledge. This reliance manifested as a reluctance to take chances in the classroom, answer questions, or engage with the reasoning behind mathematical principles.

This reliance is evident when students confront a mathematics question to which the mathematical concepts to be applied are not explicit (see figure 1.1).

This rectangular frame is made from 5 straight pieces of metal.



The weight of the metal is 1.5 kg per metre.

Work out the total weight of the metal in the frame.

Figure 1.1 – Edexcel Mathematics GCSE Summer 2017 Higher Paper 1, Question 5

While my students could grasp composite mathematical concepts, such as Pythagoras’ theorem, they often struggled to make sense of what is required by questions such as these. They resorted to approaches that are not mathematically rational, such as manipulating the numbers using any of the four arithmetical operations, or using formulae that relate to irrelevant information in the question, such as by calculating the area of the triangle. With a significant proportion of the reformed GCSE Mathematics Higher Paper requiring the application of mathematical knowledge in “a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions” (*National Curriculum in England*, 2021, paragraph 2), my students needed to develop this particular skill in order to achieve higher grades or even to pass mathematics at GCSE. Their learning practice needed to change.

1.1 The Context of this Research Study

Mathematics, especially at the secondary level, is an important subject both to schools and to students' own futures. Three government policies – Key Stage testing, first undertaken in 1991; the transfer of responsibility for school inspections to the Office for Standards in Education (Ofsted) in 1993; and the introduction of school performance tables under the governments' choice agenda in 1992 – drew attention to mathematics performance in schools. These policies have led school stakeholders, including local education authorities, to pressure mathematics faculties and teachers to raise their students' performance in the subject. The fall in England's position in the international ranking of students' performance in English, Mathematics and Science in the OECD PISA survey of Great Britain (Department for Education, 2010) led to the 2014 wholesale reform of the national curriculum, testing, and the performance indicators for school league tables. The government sought to emulate the more successful education systems of Finland, Singapore, and Shanghai (e.g. Department for Education & Truss, 2014). These policies and reforms continue to impact the culture and structure of secondary school mathematics departments as students' performance in mathematics becomes increasingly crucial for schools' survival.

1.1.1 Mathematics – An Important Secondary School Subject

In England, all 15-to-16-year-olds sit the General Certificate of Secondary Education (GCSE) at the end of their compulsory secondary school education. Children start secondary school having sat standardised English

Reading and Mathematics tests at age 11, the end of primary school. The difference between the two test scores (the progress measure), statistically calculated by the Department for Education, is used to determine students' progress in mathematics in secondary schools.

In their bid to quantify what is happening in schools and give parents more information and power regarding the choice of schools, successful governments in England have introduced and continuously improved test-based school accountability measures (Leckie & Goldstein, 2019). In 2016, As part of this measure, the government introduced the floor target "Progress 8" (DfE, 2019), replacing the previous floor target that judged a given school on the percentage of its students who achieved five C+ GCSE grades, including English and Mathematics. This new target measured individual students' progress in eight subjects from primary school national tests to their GCSE examinations. A school's Progress 8 score is the average of their students' scores presented with a 95% confidence interval. While this floor target is calculated based on students' performance across eight subjects, English and Mathematics are double-counted. A Progress 8 score ranges from -1.0 to +1.0; a score below -0.5 indicates failure to achieve the minimum standard expected by the government, and a score of +0.5 or above indicates that the students in the school are progressing above the expected level. These measures further contribute to the emphasis placed on students' performance in the particular subject of mathematics that informs teaching practices in schools and mathematics classrooms.

These accountability measures rank schools on students' attainment and progress in GCSE examinations. Before the COVID-19 pandemic began in early 2020, these rankings, in the form of “league tables”, were published in the national press, and validated a school’s reputation both in the local and national contexts. As school funding follows pupils, these accountability measures and accompanying league tables are a form of “consequential accountability” (Hanushek and Raymond, 2005), which assigns consequences to institutions that fail to meet expectations. Parental choice determines pupil intake; schools that produce positive results, therefore, become oversubscribed, while “failing” schools struggle to meet their intake quota. Poor performance also triggers an inspection by Ofsted that results in an Outstanding, Good, or Requiring Improvement rating; all schools are required to make public the full documentation of these inspections. Publicity acts as a further aspect of the control and policing of school performance, and legitimises government policy.

For students, these qualifications act as a threshold for accessing post-16 education and employment. Secondary school mathematics and its study has historically conferred positive status on students who perform well in it. It is a gatekeeper to entry into elite universities (P. Davies & Ercolani, 2019) and a higher earning power leading to economic stability (Levine & Zimmerman, 1995).

1.1.2 The Secondary School

This study took place in an inner-city London state school that enjoys a measure of popularity in the local community. The school has a cohort of 1300 mixed-gender, culturally-diverse pupils ranging in age from 11 to 16 years old. The school has enjoyed increasingly strong examination results, with a Progress 8 score of 0.3. In its last school inspection, Ofsted graded the school as Outstanding. The mathematics faculty is in a block of twelve classrooms, one of which is an ICT suite. My classroom, where the research took place, is the ICT suite. The mathematics faculty designed the Year 10 curriculum map with the intention of having students progress through the GCSE mathematics content over two years; thus, they would typically complete the program by the end of the Year 10, having started at the beginning of Year 9.

This study focuses on one mathematics class of students who were, at the beginning of the research, just commencing Year 10 studies. I chose to focus on this class as, at the time, I only taught one Year 10 class and one Year 11 class; I did not select the latter class for two reasons. Firstly, Year 11 is a shortened year, as the GCSE examinations start in May, and the students are no longer in lessons; secondly, Year 11 was planned by the mathematics faculty as a revision year, and the students would have already completed the curriculum content. I wanted the study to take place over a sufficient amount of time, and involve students learning new content.

The students in this study were 14 to 15 years old and in their fourth year of secondary education (that is Year 10), having commenced a programme of study that culminates in a series of external examinations across May and June the following academic year. The students were loosely assigned to mathematics class groups based on assumptions about their mathematical abilities, as is conventional (Boaler, 2014). This perception was based on students' performance in the internal mathematics examinations that took place at the end of the previous academic year; the mathematics faculty considered my class as of a lower-middle ability.

1.1.3 Myself: The Mathematics Teacher

From the beginning of my teaching journey, I have been aware of the potential of alternative approaches to secondary school education, having spent my formative years educated in another continent. As a Postgraduate Certificate of Education (PGCE) trainee at the UCL Institute of Education (IOE) in London, I was acutely aware of the fact that the form of UK education system was not universal, and this contributed to my initial endorsement of alternative pedagogy. Attending to these and other personal motivations and assumptions is a significant aspect of becoming a reflexive qualitative researcher; it is crucial to be faithful to the influence of my positionality on the research process and findings. Jane Miller referred to telling one's story as part of the research process, as the "autobiography of the question" (Miller, 1995, p. 23); she argued that it is a powerful validation of our experiences and their potential for rethinking teaching. To this end, in reflecting on the journey that led me to this study, I highlight two further motivations that changed my thinking as a mathematics teacher and made me consider adapting my pedagogical methods in the classroom.

1.1.3.1 Getting Expectations Wrong

In January 2008, certain events caused me to rethink the traditional pedagogical approach to mathematics that I was implementing, and particularly to question its assumptions regarding the role of the learner. It started with a student who took her GCSE mathematics examination twenty months early, in November 2007, at the start of Year 10.

Kaome (real name withheld) was one of the students in what was then my Year 10 mathematics class. My class was a “border-line” class; mathematics teachers use this term to refer to groups of students whom the faculty considers to have difficulty learning mathematics, but who, with academic support, could achieve a pass grade
C in the GCSE examination at the end of their secondary education (June 2009 for Kaome). Achieving a grade C was of great importance to schools, given the presiding government’s accountability measures, which were based on the percentage of students who achieved a grade C and above in subjects (see section 1.1.1). Kaome’s academic profile hitherto was based on her performance in the primary school Mathematics Standardised Assessment Tests (Year 6 SATs) that positioned her on entry to secondary school as of “average ability” having achieved the national expected level (Gibbs, 2011) and her performance in the Year 9 SATs that positioned her as of “border-line ability” having achieved the national expected level (Department for Children, Schools and Families, 2009)]. Based on our school’s internal statistics, 52% of students who achieved a level 5 in their Year 9 SATs achieved a C grade [or above] at the GCSE level Kaome achieved a level 5 in her Year 9 SATs.

Being “border-line”, Kaome was availed of only a limited field of mathematical concepts; she was perceived by the pedagogical authorities as lacking the cognitive capacities required to engage with higher-level concepts. Moreover, the teaching procedures even prevented Kaome and other “border-line” students from being able to explore such concepts on their own. For instance, when covering the topic of linear equations such as 2x + 7 = 15, students in “border-line” classes were only exposed to methods informed by “what is happening to x?”-style flow diagrams such as that shown in figure 1.2 below.

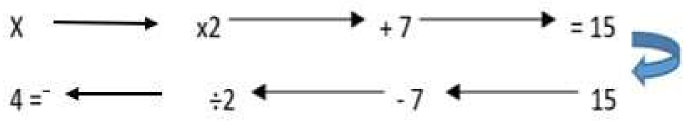


Figure 1.2 – Solving Linear Equations in Border-line GCSE Mathematics Classes

This method cannot be applied to equations such as 2x + 7 = 3x + 11, which have variables on both sides of the equal sign.

In January 2008, Kaome achieved a grade B in GCSE Mathematics. Her parents had sent her to a Saturday school in preparation for the November 2007 GCSE examinations. In personal communications, her mother informed me that the Saturday school had expected Kaome to achieve an A grade; thus, their view was that she had underachieved, while we (myself and the mathematics faculty) believed that she had over-achieved. It came down to a difference in expectations.

After achieving a B grade, Kaome moved to the higher-ability mathematics class; the faculty no longer considered her to be a “border-line” student, but now assumed that she was capable of reckoning with more advanced material. Due to a subsequent change in self-perception, she herself behaved like such a student who achieves A grades in both GCSE Mathematics and Statistics. Three years later, she went on to study medicine at university.

As a mathematics teacher, I had judged Kaome wrongly; I had relied on statistical information to limit my expectations of my students, including Kaome. In doing so, I justified to myself the restriction of the mathematics learning that I made accessible to them. As a consequence of my experience with Kaome, I decided to change this approach.

1.1.3.2 Students Taking Responsibility for their Mathematics Knowledge

As Head of Faculty, in November 2009 I decided to give all students in Year 11 the opportunity to enter their GCSE examinations eight months early. As a result of this decision, the school achieved its best GCSE Mathematics results to date, with 84% of the cohort achieving a grade C or better by the end of Year 11. In the following year, the mathematics faculty allowed all students in any secondary-level year group to enter GCSE Mathematics at a time of their choosing within the broader timeline of secondary study. Expectations for achievement became the responsibility of the individual students themselves; expectations became an index of students' beliefs about themselves and their own agency, and were no longer limited by teachers or based on past examination performance.

What became immediately noticeable to myself and my fellow faculty members was the change that took place in students' participation in their learning once they had decided to sit their GCSE examinations. The students took responsibility for what they did not know and sought to know; they became more tenacious and creative in their desire for knowledge, and supported each other's learning. Over the next four years, the faculty achieved figures ranging between 79% and 84% of students achieving a grade C or above. More students referred to themselves as “good at maths”, and, upon receiving their results in the January of the academic year, it became common for students to register to take the next set of GCSE examinations to achieve a better grade. In September 2013, however, the government began to penalise schools for early entry examinations, and our faculty stopped offering this opportunity to students.

1.1.3.3 Questioning the Taken-for-Granted

Having observed how early entry for GCSE examination challenged the taken-forgranted relationships between assessment procedures and student performance, I challenged myself to look further beyond my current thinking. Part of the learner discourse that I had initially internalised tended to link certain coordinates, such as presumed ability, ethnicity, gender, and economic profile to students’ mathematics achievement (cf Boaler et al., 2011). Subsequent independent research led me to discover that, beyond what I had seen as fact or simply assumed, other factors such as students’ perceptions of gender and ability can impede progress, especially during group work (Pozzi et al., 1993). I observed that the differential performance of ethnic minority groups is partly explained by other factors such as their attendance of lower-performing schools (Kingdon & Cassen, 2010); and that the teacher’s attitude towards characteristics such as ethnicity (positive or prejudicial) can have a significant impact on students’ participation and achievement in mathematics learning (Boaler et al., 2011). The literature confirmed what I had come to realise: that my perception of my students influenced my behaviour towards them, and, therefore, their experiences in my classroom. I decided to attempt to bring my actions in line with my expectations of the students.

I started by changing how I expected the students in my classroom to learn mathematics; I sought to restore the motivation I noticed in my students when they were able to take responsibility for the timing of their entry into GCSE Mathematics. Above all, I wanted them to make more decisions about what they wanted to learn and how they learnt it. Two years before the commencement of my doctoral research, I began pursuing this aim by giving the students in my mathematics classes the opportunity to choose the sequence in which we would learn the topics in the curriculum; I also gave them new responsibilities, asking each to prepare a mathematics topic and teach it to their peers, with the hope of bolstering their confidence in their abilities. While I sincerely believe that they had a positive effect on my students' participation, given their informal nature, I could not effectively analyse the impact of these new measures. This research study was undertaken in order to discover, with analytical clarity, the most effective means for improving the conditions of my students' engagement with mathematics.

Thus, this study aims to empower students to actively participate in all aspects of their mathematics learning in order to improve their relationship with the subject and their grasp of it. Most concretely, I am concerned to discover more effective forms of pedagogy that encourage students to apply their knowledge rationally to solve problems in the secondary school mathematics classroom; and, ultimately, that will improve their performance in GCSE examinations. To this end, I explore how "shared epistemic agency" is developed and sustained in mathematics classrooms. Shared epistemic agency, discussed in full in chapter 4, is the central concept that I have developed and used to describe and analyse students' participation in learning environments for the creation of knowledge. I propose that students with this type of agency are actively engaged in their learning, taking responsibility for what they know and do not know and acting to further their own and their peers' knowledge; if this agency is able to be sustained over a period of time, I hold that it is a powerful facilitator of the advancement of the collective knowledge of all the students in the classroom.

1.1.4 The Thesis Outline.

I have organised this study across six further chapters. In chapter 2, in which I develop my theoretical framework, I review literature on the key concepts of agency, social learning theories, pedagogy and the existing constructs of knowledge building, knowledge creation and shared epistemic agency that inform the design of the study.

This review leads me to conclude that my elaborated idea of shared epistemic agency, which embodies the six essential characteristics of the ideal learner that I have extracted from the extant literature on education theory, was the particular kind of agency required to improve the participation of the students in my classroom. An innovative pedagogy that could support the development of this agency was also needed. Students with shared epistemic agency:

intentionally act to resolve a mathematics unknowing, they seek to extend their knowledge, they explicate knowledge to each other, they take control of the learning process and as a result, they create knowledge new to them. The review revealed characterisations of shared epistemic agency in short-term classroom projects, outside a high-stakes assessment system, but these were important differences to my classroom setting.

The following research questions then emanated

1. What are the indicators of shared epistemic agency in the mathematics classroom?
2. What sustains the emergence of shared epistemic agency in the mathematics classroom?

Chapter 3 presents the qualitative action-research methodology I employed throughout the study, and the specific research design that it informed, which was developed to fit my particular aims. In this chapter, I explain the innovative pedagogy that is at the heart of this study, address the ethical issues in relation to the intervention and explain the methods of data collection.

In the chapter 4, which concerns my analytical methods, I present an original unit of analysis: an Episode of shared epistemic agency that exemplifies the objects of interest; that is, the interplay of the six characteristics mentioned above. An Episode is a snapshot of students' purposeful interactions to resolve an unknowing, hence produce knowledge new to the students. Focusing the analysis on Episodes thus allows me to select relevant moments from the hours of data.

In chapter 5, I present the findings from these Episodes. To facilitate answering the research questions and meet the aims of the study, in the first section of this chapter, I used my analysis of episodes to present a more detailed description of how the characteristics of shared epistemic agency manifested in the classroom as the students enacted the innovative pedagogy. In the second section, I elaborate on what was unique about students' epistemic interaction and I present findings that highlight how student positionings and authority impacted on the way they advanced mathematics knowledge in the classroom.

Subsequently, chapter 6 contains a discussion of the two themes that emanate from these findings in responds to the research questions. I critique the idea of shared epistemic agency as an encapsulation of the six characteristics and I propose a more holistic view of the construct. The chapter also puts forward a conceptualisation of the student and the mathematics classroom that emerged from the study and it reflects on the action research process. This reflection focusses on my role as a participants and the innovative pedagogy.

In the seventh and final chapter, I outline the contributions this study makes to the field of mathematics education, and I present a challenge to current educational policy and classroom practice. In my contribution to theory, I present my extension to the existing construct of shared epistemic agency and I indicate the extent to which this study has fulfilled its aims of participation and empowerment. My final contribution as a teacher researcher identifies the value of action research as a meta-methodology. I note the limitations of the research study and end with a call for teachers to become researchers in a bid to improve the profession.