5 FINDINGS

In this chapter, I take a closer look at the units of analysis, that is, the thirty-six Episodes of shared epistemic agency I identified from the data (see Table 4.1). Transcribing and coding each Episode allowed me to perform a more detailed consideration of how the characteristics of an Episode of shared epistemic agency manifest in the classroom. Elaborating on the Episode and on how participants interact as they direct their agency towards the characteristics of shared epistemic agency will offer insight into what is indicative of this agency and how it is sustained in the classroom. It will thereby contribute to answering the research questions.

As stated in chapter 4, an Episode of shared epistemic agency comprises three parts: Intentions, knowledge building, and New Knowledge (see Figure 5.1). An Episode starts with an Intention to resolve a state of unknowing, a lack of knowledge, and ends when the production of New Knowledge that remedies this lack (see section 4.1).

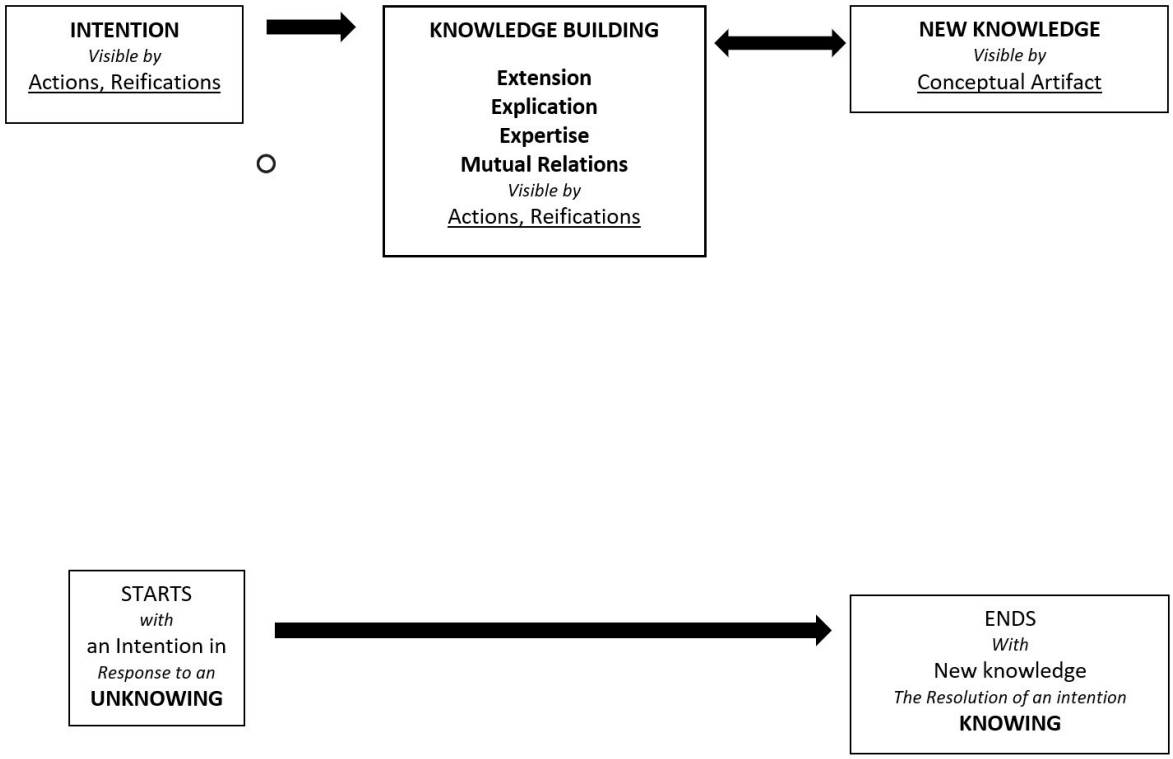


Figure 5.1 – The unit of analysis

Throughout this chapter, I will use extracts from Episodes to elaborate my findings. When explaining each discovery, I will select the Episode that I feel best demonstrates it, and I will also select Episodes based on how interesting they are and how well they show the unique personalities of the participants. While I sought to include all participants across my choice of Episodes, my priority was to explain each finding clearly. This means of selection resulted in my using extracts from sixteen out of the thirty-six Episodes. I repeated some Episodes, such as Episodes 1, 10, and 23 twice, while I used three different extracts from Episodes 2 and 6 across the chapter. The repetition of Episodes demonstrates that Episodes were complex interactions between participants that pointed to illuminate multiple aspects of participation.

The first part of this chapter will elaborate on the three parts of an Episode: Intentions, knowledge building, and New Knowledge. In the second part of this chapter, I will elaborate on participants' interactions in the classroom, their positionings during interaction, and how they expressed their authority. In this way, this chapter will highlight the findings related to each of the six characteristics that encapsulate shared epistemic agency, as well as further findings that emerge from the participants' interaction. The themes that arise from these findings will provide answers to the research questions.

5.1 Elaborating on the Unit of Analysis

An elaboration of an Episode of shared epistemic agency was made possible by the transcription and coding of each Episode. In this section I will describe a more nuanced conception of each part of an Episode. This description will identify the modes of Extension, Explication, and Expertise, elaborate on Mutual Relations, and discuss how an unknowing is resolved as New Knowledge.

5.1.1 Intentions

The Intention (see section 4.1.1) part of an Episode is the start of the Episode; it is the proactive commitment to resolve a lack of knowledge, an unknowing. An Intention orients its bearer towards any of the characteristics of Extension, Explication, or Expertise as it expresses this intentionality. Suppose the Intention relates to a participant striving to know, to extend their existing knowledge. In this case, it will orient the participant towards Extension; if it expresses their aim to make

knowledge explicit to other participants, it will orient them towards Explication; and if it expresses their consolidation of process authority, it will orient them towards Expertise. Unlike in the case of the other knowledge-building characteristics, I found no empirical evidence to show that an Intention can orient towards mutual relations. I attribute this lack of evidence to the fact that Episodes in this research are all epistemic, focusing on mathematics knowledge, while the presence of Mutual Relations is a characteristic that only indirectly supports knowledge building in this context.

An Intention can be made visible by participants' actions, that is, dialogical interactions and/or physical interaction and/or reifications (see section 2.2.2.2). It can be triggered either by a teacher participant's (TP's) or student participant's (SP's) lack of knowledge, or when a teacher participant or student participant make a judgment about an individual or group's lack of knowledge. We will call the former "Identified unknowing", and the latter "Assumed unknowing". All these elaborations of an Intention – the three ways it is made visible, in the three different orientations towards action – as well as the two types of unknowing, and the participant who initiated the Intention, will be exemplified with the extracts from four Episodes below.

Extract 5.1 - Intentions (Ext, Dialogic Interaction, Identified) - Episode 1

Context: The question $2x^2 + x - 21 = 0$ was placed on the board by teacher participants (TP) Deepz and Jevonte for the student participants to solve. This was the start of the second lesson on factorising quadratic equations. Pearl, a student participant (SP), initiated the dialogue with the question in line 1.

Part	Line	Participant	Action/Reification	Code

Intention	1	Pearl (SP)	"How can we use the same method with the x?"	I (Ext)
Knowledge Building	2	Deepz (TP)	"It's the same thing that we did yesterday."	Xpt
	3	Student A (SP)	"But what do you times together to get x?"	Ext

In this extract from Episode 1, the Intention orients Pearl towards the shared epistemic agency characteristic of Extension. The Extension orientation identifiable at line 1 that commenced the Intention was an expression of Pearl's desire to extend her existing knowledge. The Intention, once initiated, was externalised: Pearl asked, "How can we use the same method with the x?". Pearl's apprehension of her lack of knowledge triggered the Intention; thus, this Episode was triggered by an Identified unknowing.

Extract 5.2 – Intentions (Exp, Dialogical/Physical Interaction, Assumed) – Episode 9

Context: Teacher participant James is at the board introducing the concept of "less than" and "greater than" using a PowerPoint lesson prepared earlier. Student participants are focused on the board, listening to his exposition.

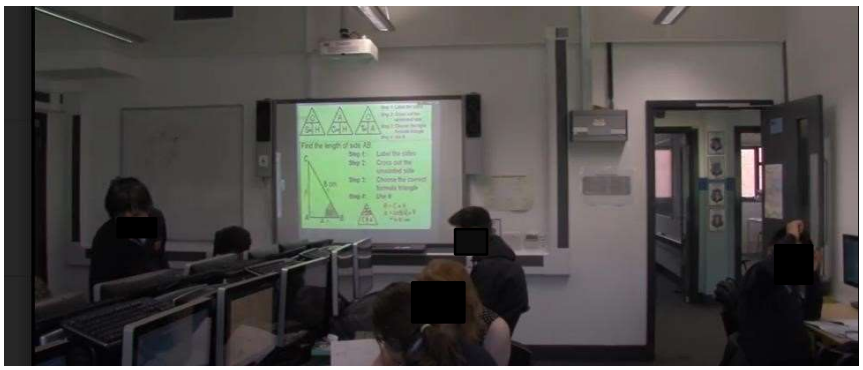
Part	**Line**	**Participant**	**Actions/Reifications**	**Code**
Intention	1	Student A (SP)	(Student A calls out from the back of the class)" James …"	MR Trust

	2	James (TP) (turning towards Student A):	"Yo"!	MR Solidarity
	3	Student A (SP):	"… I'll show you something easier?"	I (Exp)
	4	As she speaks, student A comes towards the board.		I

In this extract from Episode 9, the Intention orients Student A towards the shared epistemic agency characteristic of Explication. This orientation is because line 3, in which the Intention commenced, was an expression of Student A's desire to make the concept of "greater than" and "less than" explicit to the classroom participants. The Intention was thus initiated by Student A, a student participant, and made visible by the dialogic interaction in lines 1-3 and the physical interaction of walking up to the board in line 4. It was initiated by Student A, who assumed that a lack of knowledge existed amongst the classroom participants. Hence, this Episode was triggered by an Assumed unknowing.

Photo 5.1 – Intentions (Ext, Dialogical Interaction/Reification, Identified) – Episode 17

Context: The students are working on the questions in their booklet. Adam, a student participant, is working independently at position G. At 31:28, he raises his hand (see Photograph 5.1 below).



Photograph 5.1 – Intention (Ext, Dialogical interaction/reification, identified)

In this Photo from Episode 17, the Intention orients Adam towards the shared epistemic agency characteristic of Extension. Adam's raised hand initiated the Intention, as it was an expression of his desire for Tom, the teacher participant, to extend his existing knowledge. The Intention was made visible by Adam raising his hand, which was also a reification of the call for attention. Adam, having identified his lack of knowledge, triggered the Intention; hence this Episode was triggered by an Identified unknowing.

Extract 5.3 – Intentions (Exp, Dialogical/Physical Interaction, Assumed) – Episode 19

Context: Teacher participants Daniel and Tom are walking around the class helping students and checking their work. Daniel is using a booklet with solutions compiled by Tom. Daniel walks to Crimson and checks his work. Daniel compares Crimson's solutions to the solutions Tom has prepared.

Part	Line	Participant	Action/Reification	Code
Intention	1	Daniel (TP) (to Tom):	"Are you sure its 11.3?"	I (Xpt)
	2	Daniel walks over to Tom and puts his hand on his shoulder; they both look at the solution in the booklet, and discussion ensues. After studying their solution, they both walk back to Crimson.		MR Solidarity
Knowledge Building	3	Daniel (to Crimson):	"Did you put the 15 over 3?"	Ext

In this extract from Episode 19, the Intention orients towards the shared epistemic agency characteristic of Expertise. This orientation is because line 1, which commenced the Intention, is an expression of Daniel as a teacher participant exercising his process authority, controlling the learning process by seeking to ensure that the answers in the booklet are correct. The Intention was thus initiated by Daniel and made visible by the dialogical interaction, "Are you sure it's 11.3?", in line 1, and the physical action of walking over to Tom and placing his hand on his shoulder. It was initiated by Daniel, who assumed that a lack of knowledge was

operative in Tom's calculations. Hence, this Episode was triggered by an Assumed unknowing.

These four extracts are summarised in the table below.

Episode	Orientation	Initiated by	Visibility	Episode Trigger
1	Extension	Student participant	Dialogical interaction	Identified unknowing
9	Explication	Student participant	Dialogical interaction Physical Interaction	Assumed unknowing
17	Extension	Student participant	Physical interaction Reification	Identified unknowing
19	Expertise	Teacher participant	Dialogical interaction Physical Interaction	Assumed unknowing

Table 5.1 – Elaboration of Intentions

The elaboration of all Episodes (see Appendix 7) demonstrates that about half of the Episodic Intentions oriented participants towards Extension, followed Explication, with the lowest number of Intentions oriented towards Expertise. All Intentions that were triggered by an Identified unknowing were initiated by a student participant identifying their own unknowing, with the exception of Episode 30 (see Extract 5.18), where the student participants identified another participant's unknowing, and Episode 14 (see Extract 5.7), where a teacher participant Identified a student participant's unknowing. Episodes triggered by Assumed unknowings were initiated by both student and teacher participants.

5.1.2 Knowledge Building

The knowledge building part of an Episode is where the participants exercise their agency through their interactions to resolve the Intention. Their agency manifested as the interaction of four characteristics of shared epistemic agency: Extension, Explication, Expertise, and Mutual Relations (see section 4.1.2).

Analyses of the empirical data led me to develop a more nuanced conception of the characteristics of Extension, Explication, and Expertise from the diverse ways these characteristics of shared epistemic agency were made visible by the participants in the enactment of the innovative pedagogy. I refer to them as "modes" of each characteristic, and I classify them in the following sections. What is significant about these modes is not that they do not occur in other classrooms, but that they occur in my classroom during knowledge building as part of an Episode to resolve an unknowing.

Having completed summary sheets (see section 4.2.1.1) for each of the thirty-six Episodes, I realised that I had described how each characteristic was made visible differently on each sheet; I had written down the action or reification of the participants as observed on the video recording, and noted the diversity of expressions. I compiled all the different descriptions of the actions or reifications of each characteristic and grouped them into discrete modes to aid further analysis. Table 5.2 gives an example of four of the original descriptions of actions or reifications that I identified as indicating Extension; I have grouped these into a single mode, "Articulates unknowing".

Action and reification noted on the summary sheet	Episode	Classified mode of Extension
“That’s not what I got”	6	Articulates unknowing
Teesh asks what others got	13	
Jayzee explains knowledge limit	24	
Crimson acknowledges unknowing	26	

Table 5.2 – Classifying a mode of Extension

Having identified these modes, I designed an appropriate summary sheet (see Appendix 8), and re-watched the recordings of each Episode to identify how many times each of the modes occurred in each Episode as an indication of the mode's relevance to the research.

5.1.2.1 Modes of Extension

Extension is the characteristic of shared epistemic agency by which participants direct their agency towards striving to know in a bid to extend their existing knowledge (see section 4.1.2.1). From the analysis of the recordings, I identified five distinct modes of Extension by which participants sought to extend their existing mathematics knowledge. The five modes are as follows: Questions, Seeks affirmation, Requests, and Articulates unknowing. I identified over 170 instances of these five modes of Extension across the 36 Episodes, with Questions being the most common, followed Seeks affirmation, Requests, Challenges, and Articulates unknowing being the least (see Appendix 9).

5.1.2.1.1 Questions

Epistemic questions (referred to as Questions) are direct questions asked by a participant of another participant (who is thereby deemed to have epistemic authority, whether a teacher or student participant), or else openly presented to an audience, in a bid to extend the former's existing knowledge. These questions are usually prefaced by "why", "what", "where", "how", or "when"; they are epistemic, as they relate to mathematics knowledge. Importantly, not all sentences that have the form of grammatical questions qualify as Questions in the Episodes.

Extract 5.4 – Extension (Questions to Student Participant) – Episode 18

Context: Student participants James and Jayzee were seated next to each other. It was the beginning of the second lesson on Trigonometry. They were to finish off the booklet of questions that was started in the previous lesson. The following Knowledge building interaction commenced with James declaring his lack of knowledge.

Part	Line	Participant	Action/Reification	Code
Intention	1	James (SP):	"I do not know what to do" (Identified unknowing).	I (Ext)
	2	James (SP):	"What's ACB? … Oh, do you need to calculate angle ACB?"	Ext Question

Knowledge Building	3	Jayzee (SP) (holding up her booklet and pointing to the diagram):	"The angle is always the middle letter …"	Exp

Extract 5.5 – Extension (Questions to Teacher Participant) – Episode 29

Context: Deepz, the teacher participant, explained bounds to the classroom participants with a worked example. He then asked the student participants to copy it from the board. He asked them to ask him if "they didn't understand". At 8:56, Daniel walks up to the board and points to the working out. Teacher participants Deepz and Ty were at the teacher's table. While Daniel was asking the question, James joined him at the board. Other participants could hear the discussion.

Part	Line	Participant	Action/Reification	Code
Knowledge Building	1	Daniel (SP) (pointing at a place on the board):	"Hey Ty, where did you get this from?"	Ext Questions

	2	Deepz (TP):	"Well like, listen, you always divide by two because … look (he comes up to the board) when you are finding the upper bound or lower bound its always plus or minus five"	Exp MR Trust
	3	Daniel and James (SPs) ask further questions at the board (inaudible).		
	4	Daniel (SP):	"Ain't the answer seventy-six?"	Ext

Line 2 from Extract 5.4 and line 1 from Extract 5.5 are Questions, as they are grammatical questions that concern mathematics knowledge, and serve the purpose of extending a participant's existing knowledge. In Extract 5.5, Daniel, a student participant seeking to extend his knowledge, directed the Question to Ty, the teacher participant, whom he viewed as knowledgeable, but Deepz, the other teacher participant, assumed epistemic authority and responded to the question. In extract 5.4 line 1, the student participant James initiated the Episode with an expression of his Identified unknowing. He openly shared his unknowing with the participants who sat around him. In line 2, he directed a Question to Jayzee, the student participant who sat next to him, and Jayzee assumed epistemic authority as a knowledgeable participant. Questions are not the same as other modes of Extension that include questions such as those that request help or that are posed to seek affirmation of knowledge.

5.1.2.1.2 Seeks Affirmation

Seeks affirmation occurs when a participant seeks validation of their knowledge to ensure that they are proceeding in the correct manner. The question posed requires a 'yes' or 'no' response from a knowledgeable participant. I identified this as a mode as Extension because the participant seeking affirmation is striving to know, which in this case is evident from their desire to be confident in the knowledge they have gained.

Extract 5.6 – Modes of Extension (Seeks Affirmation) – Episode 10

Context: James, the teacher participant, is introducing the concept of representing inequalities on a number line. Student participants are joining the exposition. This Episode was initiated by James' dialogical interaction in line 1 and the reification of the number line. The Episode's Intention was triggered by an Assumed unknowing. James assumed that all or some of the student participants did not have knowledge of representing inequalities on a number line. As such, the Intention was oriented towards Explication.

Part	Line	Participant	Action/Reification	Code
Intention	1	James (TP):	"If you want to plot this here, so we know that it's less than, so we put a circle …" (Assumed unknowing)	I (Exp)
Knowledge Building	2	Student B (SP):	"… and you colour it in, right?"	Ext Seeks Affirmation
	3	James (TP):	"… yeah, you colour in the circles because its less than …"	Exp

	4	Crimson (SP):	"… and then you draw an arrow down …"	Exp
	5	James (TP):	"Then you draw an arrow down."	Exp
	6	Student A (SP):	"Wait, wait I got a question!"	MR Trust
	7	James (TP):	"Yes?"	MR
	8	Student A (SP):	"So, if its more than you draw an arrow that (pointing towards her right) across the way?"	Ext Seeks Affirmation
	9	James (TP):	"Yes."	Exp

In the dialogical interaction between the participants in Extract 5.6, Student B (in line 2) and Student A (in line 8) asks James a question by which they articulate their existing knowledge, and seek from James further affirmation of their knowledge. The questions are such that James can provide a "Yes" or "No" response (see lines 3 and 9).

5.1.2.1.3 Requests

Requests to enable Extension (referred to as Requests) are actions or reifications directed by one participant towards another participant with epistemic or process

authority, requesting an action or reification that they feel would enable the Extension of their existing knowledge; or, an action or reification directed by one participant towards another participant with authority, requesting permission to carry out an action or reification that they feel would enable the extension of their knowledge. The participant with authority could be either a teacher participant or a student participant. I have exemplified Requests in photograph 5.2 below and in Extract 5.7. In the photograph, the student participant Requests the teacher participant, who has epistemic authority, to carry out an action; while in Extract 5.7, the student participant requests the teacher participant with process authority for permission to carry out an action themselves. Requests are not direct questions about mathematics knowledge, and this differentiates them as belonging to a different mode of extension from that of Questions.

Photograph 5. 2 – Modes of Extension (Requests) – Episode 31

Context: Teacher Participants Pearl and Jayzee stand in front of the class, explicating the concept of finding the area of any triangle. They were explicating how to label a triangle's sides and angles in response to Crimson seeking affirmation. Roan raised his hand at 4:50.

Photograph 5.2 – Modes of Extension (Requests)

Raising his hand, in this context, was a reification of a request for the Extension of knowledge. Roan wanted either of the teacher participants, whom he viewed as epistemic authorities, to come over to him and help with an aspect of the topic. The field of Requests, as a mode of Extension, does not include all grammatical requests made by participants, but is specific to requests to enable Extension.

Extract 5.7 – Modes of Extension (Requests) – Episode 3

Context: The question $2x^2 + x - 21 = 0$ was placed on the board by teacher participants (TP) Deepz and Jevonte for the student participants to solve. The previous lesson was on factorising quadratic expressions. Teesh, a student participant, initiated the Episode. The Episode's Intention was triggered by her Identified unknowing. She knew how to factorise quadratics, but did not know how to solve them. As such, the Intention was oriented towards Extension, as she sought to extend her knowledge to the solving of quadratic equations. This was the basis of her dialogical interactions.

Part	Line	Participant	Action/Reification	Code
Intention	1	Teesh (SP):	"Let me do it on the board" (Identified unknowing).	I (Ext) Requests
Knowledge Building	2	Deepz (TP):	"No, but that's not it though."	Xpt
	3	Teesh (SP):	"Ok, but then when I do it, I show you what I can do …" (inaudible).	Ext Requests
	4	Deepz (TP):	(Inaudible response.)	
	5	Teesh (SP):	"I will do what I can do then …"	Ext Requests
	6	Deepz (TP):	"No but …" (inaudible).	
	7	Teesh (SP):	"That what I said, I will do what I can do then you do the rest."	Ext Requests
	8	Deepz (TP):	"Ok, come up and do it" (Deepz starts to prepare the board for Teesh to write on).	Xpt Controls Manages

In line 1 of the extract, Teesh, the student participant, Requests of Deepz, the teacher participant with process authority, to be allowed to come to the board and extend her knowledge by showing what she knows. The Requests to extend her knowledge continued in lines 3, 5 and 7. The action that Teesh felt would extend her knowledge was her coming to the board and publicly starting to factorise the

quadratic equation. She was confident that knowledgeable participants would explain how to proceed with solving the quadratic equation, and she trusts that she will extend her knowledge in the process.

5.1.2.1.4 Challenges

Epistemic challenges (referred to as Challenges) occur when a participant challenges the veracity of knowledge presented to them by another participant with epistemic authority, in a bid to extend their existing knowledge. In this mode of Extension, though the participants seek to extend their existing knowledge, their current knowledge is sufficient to challenge the knowledge presented to them, though it requires Extension to move beyond its current point.

Challenges are coded as a mode of Extension where their essence in context is to extend knowledge. Challenges A and B below show the difference between a challenge coded as Extension and a challenge coded as Expertise (see section 4.1.2.3).

Extract 5.8 – Modes of Extension (Challenge A) – Episode 2

Context: The participants were engaged in a discussion regarding factorising the equation $2x^2 + x - 21 = 0$. Crimson, a student participant, seeks from Jevonte, the teacher participant, confirmation of the procedure for factorising quadratic equations. I selected this extract for the ensuing discussion where Pearl, another student participant, put forward the suggested solution of the numbers 6 and 7, but was still unsure about the negative numbers. This extract shows her challenging the logic of what Crimson was saying.

Part	Line	Participant	Action/Reification	Code
Knowledge Building 4:30	1	Crimson (SP):	"So it has to add to make minus fortytwo, so it will be minus six."	Exp
	2	Pearl (SP):	"Not add."	Ext Challenges
	3	Crimson (SP):	"it's minus six, minus six plus one equals minus six."	Exp
	4	Pearl (SP):	"What!"	Ext Challenges
	5	Crimson (SP):	"Add minus six plus seven."	Exp
	6	Pearl (SP):	"Minus six plus seven doesn't give you forty-two, though!"	Ext Challenges

Extract 5.9 – Modes of Extension (Challenge B) – Episode 14

Context: Tom the teacher participant, was about to show the class how to solve simultaneous equations involving a plus and a minus ($6x - 3y = 3$ and $x + 3y = 11$).

Student A, a student participant, asked if she could do the question on the board. Tom ascertained her knowledge first, then let her work the question out on the board. As part of her procedure to eliminate y, she added the two equations. Tom walked up to her and whispered line 1 of the dialogue below at 21:32.

Part	Line	Participant	Action/Reification	Code
Intention	1	Tom (TP):	"You made a mistake; you're supposed to take away." (Identified unknowing)	I (Xpt)
Knowledge Building	2	Student A (SP):	"No, you're not; this (pointing to working out) will give you a minus."	Exp A challenge
	3	Tom (SP):	"No, it won't."	Xpt

In the context of extract 5.8 (challenge A), Pearl was seeking to extend her existing knowledge, and Crimson had taken up epistemic authority. Pearl Challenges Crimson's knowledge in line 2, 4, and 6. This dialogical interaction that culminates in line 6 shows how Pearl Challenges the veracity of what Crimson told her in seeking to extend her knowledge.

Challenge B (see Extract 5.9), in contrast, does not highlight a mode of Extension. The dialogical interactions performed by Tom, the teacher participant, initiated the Episode; his Intention was triggered by what he identified as Student A's unknowing. He was not seeking to extend his existing knowledge when he challenged Student

A's solution on the board. He was taking up his process authority as a teacher participant to check the quality of the knowledge that Student A sought to Explicate to the classroom participants. Student A responded with a further challenge to Tom in line 2. This challenge was an Explication, as student A attempted to make the knowledge more explicit to Tom and was not seeking to extend her own existing knowledge.

5.1.2.1.5 Articulates Unknowing

Articulates unknowing occurs when a participant identifies their lack of knowledge in a bid to have a knowledgeable participant extend their existing knowledge. I highlight this mode of Extension in the extract from Episode 6 below.

Extract 5.10 – Modes of Extension (Articulates Unknowing) – Episode 6

Context: Teesh, the teacher participant, was at the board showing other participants how to use the quadratic formula to solve the questions she had posed to the class. Crimson, a student participant, was calling out the answer, and Teesh was writing the answers on the board. At 14:39, Student A, a student participant, made a dialogical interaction.

Part	Line	Participant	Action /Reification	Code
Knowledge Building	1	Student A (SP) (to the class):	"I didn't get that, but I got the same calculation in my calculator."	Ext Articulates Unknowing

	2	Jayzee (SP):	“What did you get?”	Xpt
	3	Student A (SP) (to Jayzee):	“I got ...”	Ext Articulates Unknowing
	4	Student B (SP):	“Are you sure, student A, because it happened last time …”	Xpt
	5	Teesh (TP):	“Everyone got this, yeah?”	Xpt
	(Classroom chatter.)			
	6	Student A (SP) (to Teesh):	“I didn’t get it.”	Ext Articulates Unknowing
	7	Crimson (SP):	(Turning to Student A with surprise) “Oh, you didn’t? What did you get?”	Xpt
	8	Student A (SP):	“I put this in my calculator” (she passes her calculator to Crimson, who studies it).	Ext Articulates Unknowing
	9	Teesh (TP) (to the class):	“So, who got the one with the minus then?”	Xpt
	10	Crimson (SP) (to Student A):	“You did two minuses, Student A.”	Exp MR

	11	Crimson (SP) (to Student A):	“It is not minus; it’s ordinary five.”	Exp

Student A sought to extend her knowledge in Extract 5.10 lines 2, 3, 6 and 8 by publicly articulating her lack of knowledge, expecting this to trigger another participant to act to extend her existing knowledge.

Not the articulation itself, but the demonstration of confidence that the statement will trigger another participant to act, marks it as a mode of Extension. As Teesh was writing the solution to the quadratic equation on the board, line 1 was spoken loudly in the presence of the whole class, directed to no one in particular and everyone in general. Student A made the statement with the subjective belief that it would trigger a knowledgeable participant in the class to act to extend her knowledge.

5.1.2.1.6 Summary

The five modes of Extension by which participants sought to extend their mathematics knowledge are summarised in Table 5.3 below.

Mode of Extension	**Description**
Questions	• Epistemic • Directed towards a knowledgeable other • Usually prefaced by “why”, “what”, “where”, “how”, or “when”

Seeks affirmation	• Participant seeks to confirm existing knowledge • The question posed requires a “yes” or “no” response
Requests	• Participant requests an action or reification from another participant, or • Participant requests to be allowed to carry out an action or reification • Directed towards a participant with authority
Challenges	• Participant challenges the veracity of another’s’ epistemic authority • Not to be confused with its function as a mode of Expertise (Challenge B)
Articulates unknowing	• Participant articulates their unknowing • Directed towards knowledgeable participant(s)

Table 5.3 – Modes of Extension Summary

5.1.2.2 Modes of Explication

Explication is the characteristic of shared epistemic agency by which participants direct their agency towards making mathematics knowledge explicit to another participant or group of participants (see section 4.1.2.2). The knowledgeable participant assumes epistemic authority if they decide to Explicate mathematics knowledge to another participant, and they have authority bestowed upon them if another participant asks them to Explicate mathematics knowledge. I elaborate on this distinction in section 5.2.1. From the analysis of the recordings, I identified four distinct modes of Explication employed by the participants in this research: Clarifies,

Affirms, Tells, and Explicates unknowing. I identified over 200 instances of Explication across the 36 Episodes, Clarifies being the most occurring mode, followed by Affirms, Tells, and Explicates unknowing being the least occurring (see Appendix 9).

5.1.2.2.1 Clarifies

Clarifying knowledge for another (referred to as Clarifies) occurs when a participant acts to help another participant make meaning of a mathematics concept during knowledge-building interactions. The extract from Episode 23 below is an example of this mode of Explication.

Extract 5.11 – Modes of Explication (Clarifies Knowledge) – Episode 23

Context: This was the second lesson on composite functions. The previous lesson ended with participants working out solutions to the questions in their booklets on the board. At the start of this lesson, as a continuation of the previous day's lesson, teacher participant James calls on Crimson, a student participant, to come to the board and show the class of participants how to solve a question on composite functions.

Part	Line	Participant	Action/Reification	Code

Knowledge Building	1	Crimson (SP) (writing on the board):	"So, when you're given this question, you always look to the one here (pointing) to the left."	Exp Clarifies
	3	Student A (SP):	"Yeah …"	
	4	Crimson (SP):	"So, if its gf you will look at the g because it's the one on the left, ok … always remember that, so whenever you get a question like this you want to find out what's here (pointing) in this case it's f, so we	Exp Clarifies
			know straight away that we will be using g …."	
	5	Student A (SP):	"Yeah …"	
	6	Crimson (SP):	"… and putting it here …"	Exp Clarifies

In this extract, lines 1, 4, and 6 reveal Crimson's actions, including the reification of the mathematical working out on the board, and his dialogical interaction is aimed at helping the other participants to make meaning of the concept of composite functions. This action and reification by Crimson is to support the advancement of the mathematics knowledge of participants in the learning community.

5.1.2.2.2 Affirms

Affirms occurs when a participant acts to affirm the mathematics knowledge of another participant, to support the advancement of the other participants' existing mathematics knowledge. In the data from the thirty-six Episodes, Affirms always occurred in response to another participant seeking to extend their mathematics knowledge by seeking affirmation. Extract 5.6 from Episode 10, presented in section 5.2.1.5 above, is an example of the emergence of this mode. In lines 2 and 8, Student B and Student A sought to extend their existing mathematics knowledge by seeking affirmation from James, the teacher participant. In lines 3 and 9, James affirms their knowledge in a dialogical interaction. This interaction serves to affirm Student A and Student B's existing knowledge; in this way, James advanced the quality of their mathematics knowledge by remedying their uncertainty.

5.1.2.2.3 Tells

Tells as a mode of Explication occurs when a participant offers up mathematics information as a statement to other participants to support existing mathematical knowledge, without explaining the mathematical principles that underpin the information, as exemplified in the extract below. This extract is again from Episode 6, suggesting the complexity of epistemic interactions in each Episode.

Extract 5.12 – Modes of Explication (Tells) – Episode 6

Context: Teesh, the teacher participant, showed participants how to use the quadratic formula to solve the question she had posed to them. Crimson, a student participant, was calling out the answers, and she was writing it on the board. The rest of the classroom participants were engaged in comparing their work to what was written on the board. This Episode was initiated by Teesh, the teacher participant, and the Intention was triggered by her Assumed unknowing. She Assumed that all or some of the participants did not have the knowledge required to solve the question she had posed to them. The Intention oriented towards Expertise, as Teesh sought, through the dialogical interaction, to control how Crimson explicated his knowledge.

Time	Line	Participant	Dialogue	Code
Intention	1	Teesh (TP):	"So, Crimson, what did you…?" (Assumed unknowing)	I (Xpt)
Knowledge Building	2	Crimson (SP):	"Do you want me to say the whole thing?"	
	3	Teesh (TP):	"Huh?"	
	4	Crimson (SP):	"Do you want me to give you the equation?"	
	5	Teesh (TP):	"Yes, tell me how you wrote it."	Xpt

	6	Crimson (SP):	“Minus five…”	Exp -Tells
	7	Teesh (TP):	“Minus five …” (writing on the board).	Xpt
	8	Crimson (SP):	“Plus five squared.”	Exp - Tells
	9	Teesh (TP):	“Plus, five squared” (writing on the board).	Xpt
	10	Crimson (SP):	“Plus, and then the square root” (gestures square root in the air).	Exp - Tells

In this extract, Crimson, in lines 6, 8, and 10, offers up mathematics information as a statement to the teacher participants to support the advancement of the mathematics knowledge of the classroom participants, without explaining how or why he has arrived at this information. However, in the context of the Episode, Teesh, the teacher participant, repeats what Crimson says and shows the working out on the board, publicly checking and confirming the mathematics knowledge.

5.1.2.2.4 Explicates Unknowing

Explicates unknowing occurs when a participant makes explicit the unknowing of another participant to support the advancement of the other participants’ existing mathematics knowledge. I exemplify this mode in the extract from Episode 1, which I repeat below.

Extract 5.13 – Modes of Explication (Explicates) – Episode 1

Context: The question $2x^2 + x - 21 = 0$ was placed on the board by teacher participants Deepz and Jevonte for the student participants to solve. This action was at the start of the second lesson on factorizing quadratic equations. Pearl a student participant initiated the Episode with the dialogue interaction in line 1.

Part	Line	Participant	Action/Reification	Code
Intention	1	Pearl: (SP):	"How can we use the same method with the x?" (Identified unknowing)	I (Ext) Questions
Knowledge Building	2	Deepz (TP):	"It's the same thing that we did yesterday."	Xpt
	3	Student A (SP):	"But what do you times together to get x?"	Ext Questions
	4	Deepz (TP):	"You do twenty-one times minus two equals minus forty-two."	Exp Clarifies
	5	Pearl (SP):	" No, no … you see how we split it; what do we split the x?"	Ext Questions
	6	(Inaudible chatter between Teesh, Pearl, and Student A)		MR

	7	Teesh (SP):	“Oh, I see what you mean … Deepz, you know what she’s trying to say? You see how there’s usually a number in the middle; she’s saying, how do you split it if there’s only an x?”	Exp Explicates Unknowing
New Knowledge	8	Jevonte (TP):	“There’s a one in front of it.”	Exp Clarifies
		Deepz (TP):	“So, it’s one basically; x is one.”	

In this extract, Pearl, in lines 1 and 3, Questions to extend her existing knowledge. In line 4, Deepz, the teacher participant, Clarifies in response to Pearl. However, this Explication received by Pearl did not extend her existing knowledge, as neither Deepz nor any of the other participants engaged in the epistemic interaction offered the desired Explication that would enable Pearl to solve the quadratic equation. Pearl wanted to know the coefficient of the x in the equation. She did not realise that the coefficient of the x was one. It took Teesh in line 7 to make Pearl’s unknowing explicit, and it was after Teesh had made this unknowing explicit that Jevonte, the teacher participant, could resolve the unknowing in line 8.

5.1.2.2.5 Summary

The four modes of Explication by which participants make their mathematics knowledge explicit to another participant are summarised in Table 5.4 below.

Mode of Explication	**Description**

Clarifies	Makes mathematics knowledge meaningful for another
Affirms	Affirms the knowledge of the participant who Seeks affirmation General response is “Yes” or “No”
Tells	States mathematics information Does not explain the mathematics underpinning the information
Explicates unknowing	A participant makes another’s unknowing explicit. Aids the advancement of another’s knowledge

Table 5.4 – Modes of Explication Summary

5.1.2.3 Modes of Expertise

Expertise is the characteristic of shared epistemic agency by which participants direct their agency towards expressing process authority (Oyler, 1996 p. 6) in the classroom community. The participant takes control of the learning culture of the classroom (see section 2.3.2), including of how the learning is to take place and of the learning behaviours of the participants. The three distinct modes of Expertise employed by the participants are Controls learning behaviour, Checks current knowledge and Manages learning resources. These will be referred to as Controls, Checks and Manages for brevity. I identified over 130 instances of these three modes of Expertise across the 36 Episodes, with Controls occurring the most, followed by Checks, and Manages being the least occurring (see Appendix 9).

5.1.2.3.1 Controls

Controls learning behaviour (referred to as Controls) occurs when a participant assumes authority over how knowledge is advanced in the classroom community, including how other participants behave, to ensure that the mathematics knowledge of all participants is advanced. This authority includes such functions as controlling the pacing and sequence of the lesson in line with the pre-prepared lesson plan. For example, in Episode 8, Teesh said, “I’m going to start moving on because you people are taking long” (27:41). Teesh, the teacher participant, wanted the class to finish solving the question on the board independently so that they could go over it together as a class. The statement shows her in control of the pace.

In other Episodes, such as in Episode 1 Extract 5.1 above, in line 2, Deepz responded to Pearl’s Extension with the statement “it’s the same thing that we did yesterday.” By this statement he was attempting to control Pearl’s learning process. The statement reified what the school considered good practice; that is, Deepz encouraged her to go back over the previous day's work and make an effort to remember what she had learnt previously. A similar example is Episode 6 Extract 5.10 above: in line 5, Teesh, the teacher participant, controlled how Crimson presented his mathematics knowledge to the class. In this way, she controlled the learning behaviour.

5.1.2.3.2 Checks

Checks current knowledge (referred to as Checks) occurs when a participant inspects the current mathematics knowledge of another participant in order to ensure

that the process of knowledge advancement is taking place. This is exemplified in the extract from Episode 25 below.

Extract 5.14 – Modes of Expertise (Checks) – Episode 25

Context: This is the second lesson on algebraic functions. Jayzee And Beyoncé are seated next to each other, working on composite functions. This extract from the Episode shows how Beyoncé takes on responsibility for Jayzee's knowing, by checking Jayzee's current knowledge at each stage of her Explication.

Time	Line	Participant	Action/Reification	Code
Knowledge Building	1	Beyoncé (SP):	"I don't know if this is right, but this is what I did" (she puts her booklet in between them and points to her working out).	Exp Clarifies
	2	Beyoncé (SP):	"You see how x is first" … (she pauses) ... "yeah?" (she looks up at Jayzee).	Exp Clarifies. Xpt Checks
	3	Jayzee (SP):	(Nods her head showing agreement.)	

	4	Beyoncé (SP):	(Beyoncé continues with her explanation) “and this is second …” (she looks up at Jayzee again).	Exp Clarifies
	5	Jayzee (SP):	(Nods in agreement.)	
	6	Beyoncé (SP):	“…yeah?”	Xpt Checks
	7	Beyoncé (SP):	“in this one, x is first” (she pauses and looks at Jayzee, who does not nod in agreement).	Exp Clarifies Xpt Checks
	8	Beyoncé (SP):	“You see how x is first?” (she pauses and looks at Jayzee, Jayzee doesn’t nod in agreement).	Exp Clarifies Xpt Checks
	9	Beyoncé (SP):	“What don’t you get?”	Xpt Checks
		(Explanation continues.)		
	11	Beyoncé (SP):	“What don’t you get, like where?”	Xpt Checks
	12	(Explanation continues.)		
	13	James (TP) walks up to them and stands behind, listening to Beyoncé’s (SP) explanation.		Xpt Checks

	14	Beyoncé (SP) (to James):	“Am I right?”	Ext Seeks Affirmation
	15	James (TP) nods in agreement and continues to listen.		Exp Affirms
	16	James (TP) walks away to another student.		Xpt

In this extract, Beyoncé, in lines 2 and 6, checks Jayzee’s knowledge using the dialogical interaction of “yeah?”, and awaits a response from Jayzee. The word “yeah”, with an interrogative tone, was used by Beyoncé to check that the knowledge advancement due to her Explication was taking place. In lines 4, 7, and 8, Beyoncé checks Jayzee’s knowledge by the physical action of looking at her and waiting for a nod. The physical action of looking at Jayzee is a reification of the implicit phrase “yeah, does the explanation make sense to you?”. In lines 9 and 11, Beyoncé explicitly asks what Jayzee does not understand.

5.1.2.3.3 Manages

Managing learning resources (referred to as Manages) occurs when a participant manages the resources that help advance mathematics knowledge in the classroom community. The resources include the concrete learning resources such as the interactive whiteboard, PowerPoint lesson plans, equipment such as worksheets and booklets, and human resources such as myself, the classroom teacher. This mode of Expertise demonstrates how the teacher participants direct their agency towards

utilising resources to advance community knowledge. This mode of Expertise is not commonly observable within an Episode, as it involves processes external to the lesson that set up the learning. The extract below that exemplifies the mode is taken from the start of a lesson and not an Episode.

Extract 5.15 – Modes of Expertise (Manages) – Recording 7

Context: This takes place at the beginning of a lesson. Crimson, the teacher participant, arrives before the other participants and sits at the teacher's table. Beyoncé, the second teacher participant, along with another participant, arrived next, stood by the teacher's table, and proceeded to engage in conversation while others walked directly to their usual seating positions and sat down.

Time	Line	Participant	Action/Reification	Code
1.22	1	Me:	"Is there a reason why there is a delay? Can I do something?"	
	2	Crimson (TP):	"No, no, no, it's fine, its fine"	Xpt– Controls
	3	(I walk up to the teacher's desk, and conversation ensues between Crimson and me.)		
	4	Crimson (TP):	"All right, guys, can you get your books out, your green book and your normal book."	Xpt – Controls

	5	Deepz (SP):	“Don’t we need a booklet or something?”	
	6	Crimson (TP) (to Daniel, who had been	“You sit down …”. (Inaudible. Daniel goes to sit down.)	Xpt – Controls
		standing at the teacher’s desk):		
	7	Crimson (TP):	“Ms Mezue is going to hand out the booklets.”	Xpt – Manages
	8	Me:	“No, everyone has their booklets.”	
	9	Crimson (TP):	“Everyone has their booklet.”	
	10	Deepz (SP):	“I handed mine out to someone yesterday.”	
	11	Crimson (TP):	“Well, sorry, everyone has their booklet if you don’t have yours well …”	Manages
	12	(Crimson manipulates the wall plug and wires in a bid to get the white board to function.)		Xpt – Manages

	13	Deepz (SP):	“I wasn’t given a booklet	
	14	Crimson (TP):	“What do you mean you weren’t given a booklet?”	
	15	Deepz (SP):	“cos I wasn’t here last Friday, remember last Friday …	
	16	Crimson (TP):	“Well, you have to ask Ms. Mezue.”	
	17	Crimson (TP):	“Get your green books and your normal books out?”	Xpt – Controls
	18	Deepz (SP) (to me):	“Miss, I wasn’t here on Friday when they gave out the booklet; I wasn’t here last Friday.”	
3:26	19	Crimson (TP):	“Copy down the title.”	Xpt – Controls
	20	Pearl (SP):	“Just the title?”	
	21	Crimson (TP):	“If you want to copy down the others …”	Xpt – Controls
	22	Student A (SP):	“Crimson, Can I have paper, please?”	
	23	Crimson (TP):	“Paper” (he gives her paper).	Xpt – Manages
6:00	24	(I return to the class with a booklet.)		

	25	Crimson (TP) (to me):	“Miss, can you sit down please and do the work.”	Xpt – Manages
	26	(I give the booklet to Deepz and sit down. While Crimson goes around to check on the questions, the participants are working on from the board.)		Xpt – Checks

In this extract, Crimson, in lines 11, 12, 23, and 24 manages the concrete resources (booklets, paper, the whiteboard) necessary for the learning process to take place in the mathematics classroom. In lines 7 and 25, he manages the human resources (me). In line 7 he directs me to give out the booklets and in line 25 he directs me to sit down and do the work as a student participant. Other examples of participants managing concrete resources to enable the process of learning include in Episodes 3 and 4, in which the teacher participant prepares the whiteboard so that a student participant can show the class how to solve a mathematics question. In Episode 11, in which the teacher participants James and Adam each manage the PowerPoint when the other is explicating a mathematics concept to the classroom community, is another example. Lastly, a further example is evident in Episode 14, in which student participant Pearl assumed the role of teacher participant and supported Tom with his PowerPoint presentation, as his partner Beyoncé was absent on the day.

5.1.2.3.4 Summary

The three modes of Expertise, by which participants directed their agency towards expressing process authority in the classroom community, are summarised in Table 5.5 below.

Mode of Expertise	Description
Controls	• Controls how participants learn such as; ○ The pace of learning ○ Participants learning process
Checks	• Checks participant's current knowledge
Manages	• Manages concrete resources • Manages human resources

Table 5.5 – Modes of Expertise Summary

5.1.2.4 Mutual Relations

Mutual Relations is the characteristic of shared epistemic agency that highlights the ways participants channel their agency towards relating with other participants in the classroom community. Actions and reifications coded as Mutual Relations can be contextual or non-contextual, as well as being conducive or non-conducive to the advancement of mathematics knowledge

5.1.2.4.1 Mutual Relations as Contextual or Non-contextual

The identification and interpretation of Mutual Relations requires an internal perspective on the context and the participant expressing it. As an ethnographic participant observer, I bring to this part of the study my awareness of the flexible interpersonal relationships in my classroom. Various actions and reifications can correspond to the same Mutual Relation, while the same action or reification can correspond to multiple distinct relations. For example, in Episode 9 (see Extract

5.2), where I considered the actions and reifications of James to be solidarity in line 2, the physical interaction of James turning towards Student A, and the dialogical interaction of the word “Yo!” shows James’ solidarity with Student A. Using the informal and affectionate address “Yo!” meant that even though James was the teacher participant, and Student A was the student participant, they were both participants in learning mathematics. This act of solidarity encouraged Student A to come forward and make a contribution knowledge to the community’s knowledge.

A participant could demonstrate solidarity through other actions or reifications, such as in Episode 19 (see Extract 4.1), in which Daniel’s physical actions, described in lines 2 and 14, of placing his hand on Tom’s shoulder also showed solidarity within the context of that Episode and the relations Daniel had with Tom, regardless of the fact that Daniel was inferring, in line 2, the possibility that Tom might be incorrect in his mathematics solution. The placing of his hand on his shoulder acted to soften the dialogical interaction in line 1, allowing both teacher participants to resolve the unknowing.

These actions and reifications are Individual and contextual because it was not just the hand on the shoulder that identified the Mutual Relation; the context of the action was part of the identification, as placing the hand on a participant's shoulder could also be viewed as an act of aggression in another context and between different participants.

Trust was also demonstrated in various ways across episodes. For instance, in Episode 9 (see Extract 5.2), in line 1, while James, the teacher participant, is introducing the lesson, Student A calls out from the back of the class. The dialogical interaction of the call showed the relation of trust between Student A and James.

Student A trusted that she could call out and be listened to. The strength of this trust is recognised in relation to the conventional classroom that Student A and James experience in most other subjects. In these classroom environments, students seek permission to speak out or to make a contribution. In contrast, Student A calls out from the back of the class and starts to walk towards the front of the class even before James responds. This trust enabled her to proceed to initiate an Episode; in this way, it was conducive to the advancement of knowledge. Trust was also shown by the actions of James and Daniel in Episode 29 (see Extract 5.5). In line 3, both student participants come up to the board to extend their mathematics knowledge.

Daniel had directed his agency toward Extension as he Questions Deepz in line 1. When Deepz, the teacher participant, went to the board to explicate the mathematical concept of bounds, Daniel and James followed him. This physical interaction showed their trust in the learning community, that they could act in whatever ways they needed to in order to extend their mathematics knowledge, including going up to the board without the permission of the teacher participant. Their trust was conducive to the advancement of their mathematics knowledge and that of the other participants who were listening.

Not all Mutual Relations were contextual to the participants and the Episode. For example, in Episode 2 (see Extract 5.17), after Crimson Tells in line 5, Teesh responds with the dialogic interaction “Smart. It is!” This constitutes a positive reinforcement, directed at a member of the learning community, that would hold the same meaning in any Episode with any of the participants. Similarly, in Episode 30

(see Extract 5.18), in line 14, Deepz' dialogical interaction "Everyone makes mistakes, that's why we're here" is a motivational message that holds the same meaning regardless of the context.

Nevertheless, as a result of the contextual nature of most actions and reifications coded as Mutual Relations, I did not see fit to categorise the actions and reifications that made this characteristic visible into modes.

5.1.2.4.2 Mutual Relations as Conducive or Non-Conducive for the Advancement of Knowledge

Mutual Relations are said to be conducive when the corresponding actions and reifications contribute to the advancement of mathematics knowledge, and nonconducive when they do not contribute to the advancement of mathematics knowledge.

The previous section addressed how Mutual Relations can be conducive to the advancement of mathematics knowledge. An illustration of how Mutual Relations can be non-conducive can be found in Episode 19 (see Extract 4.1). As Daniel, Tom, and Crimson interacted in lines 2-10, Roan, who was seated next to Crimson, had been listening. In line 11, Roan stood up and made a comment. Daniel responded in line 12 with the dialogical interaction, "I wasn't talking to you, sit back down". This utterance is rude in any context, and reveals the presence of undesirable Mutual Relations that are non-conducive to the advancement of mathematics knowledge, as it clearly prevented Roan from offering a contribution. Table 5.6 below outlines the Mutual Relations that I identified across Episodes, and the total number of observed actions and reifications corresponding to Mutual

Relations that were conducive or non-conducive to the advancement of mathematics knowledge.

Mutual Relations conducive to knowledge Advancement	**Mutual Relations non-conducive to knowledge advancement**
Equity, solidarity, persistence, respect, empathy, trust, helpfulness, confidence acknowledgement	Rudeness, anger, frustration, rejection, distraction, disrespect
Total across Episodes: 90	Total across Episodes: 32

Table 5.6 – Classification of Mutual Relations

5.1.3 New Knowledge

I coded, as New Knowledge, the part of an Episode wherein mathematics knowledge emerges from knowledge building as a resolution of the unknowing that triggered the Intention. Given that it resolves an unknowing, this knowledge is new to the participants involved in the Episode, and is considered legitimate if it is able to resolve the unknowing for all participants involved in the Episode. I consider two issues in this section that arise from this fact, and which highlight the rigour of this study and its participants' agency. These two issues are: how the end of an Episode is indicated and how the New Knowledge is built. I note that it is not the emergence of the New Knowledge that brings the episode to an end, but rather, it is the participants acknowledging that the New Knowledge has resolved an unknowing, thereby advancing their mathematics knowledge, that brings the Episode to a close.

I tracked back through each episode to find the source of the New Knowledge and observed that it results from the participants appealing to the mathematical principles themselves or to a knowledgeable participant.

5.1.3.1 Acknowledging the resolution of an episode

Acknowledging the resolution of an Episode takes the form of action and/or a reification. In Extract 5.13 (see section 5.1.2.2.4), the Episode ended with lines 8 and 9, when Jevonte Clarifies and resolves Pearl's unknowing with the dialogic contribution, "There is a one in front of it", and when Deepz confirmed this in line 9 with the statement, "So it's one basically". This brought the Episode to an end, as Pearl and Student A then had the knowledge required to solve the quadratic equation. They did not direct their agency towards further Extension, but rather proceeded to solve the equation. In Episodes in which more participants were involved, all participants acknowledged that the unknowing has been resolved to bring the Episode to an end. For example, in Extract 4.2 (see section 4.1.2.4), the Episode entered its final stages when Deepz, in line 10, said "Oooh, that's smart", expressing his appreciation of Student A's New Knowledge. This was followed by other participant's dialogic interactions, "Ah", in line 11, expressing their acknowledgement of the New Knowledge. However, the Episode does not end until line 18, when the teacher participant Checks by asking, "Everyone understands that?", and, on receiving acknowledgement in line 19, resumes his explanation. He had paused this to allow Student A to present her New Knowledge, and to confirm that the rest of the participants shared this knowledge too.

Thus, acknowledging the impact of New Knowledge, the end of an Episode can be seen to occur when participants no longer direct their agency towards further

Extension. This is also exemplified in Episode 6, in which the Intention was triggered by an Assumed unknowing (see Extract 5.12). Teesh, the teacher participant, was consequently presenting the step-by-step process for solving quadratic equations using the quadratic formula to all participants. From the extract of the end of this same Episode (see Extract 5.16 below), the Episode came towards its conclusion in line 36, when student participant Crimson Tells a value of x. In line 35, Teesh then

Checks with the dialogic Interaction, “Who else got this?”. The Episode ends in line 42, when no further participants direct their agency towards Extension, indicating that all unknowing has been resolved. A new question was then placed on the board for all participant to try.

A reification, such as clapping, can indicate participants acknowledging the New Knowledge, thus the resolution of the Intention staged in Episode 11 (see section 5.2.3.2). The Episode neared its conclusion when participants recognised the connections between solving linear equations and solving linear inequalities and several participants said, “ah” which reified the resolution of their unknowing. This was followed by some participants clapping, some saying “ok”, and Student A, who taught an earlier lesson on solving linear equations, saying, “That’s what I taught you guys”. The Episode ends when no participant directs their agency towards Extension. The teacher participants then moved onto a slide with new inequality problems for the participants to solve.

5.1.3.2 Building New Knowledge as Dimensions of Appeal.

New Knowledge is developed through participants' interactions during the knowledge-building phase of an Episode. I tracked back through the whole of each Episode to find the source of the New Knowledge; I ascertained that it is concretely produced during knowledge-building by participants' appeals to conceptual knowledge, a knower, or procedural knowledge. These appeals could be viewed as dimensions, as some Episodes involve a combination of two or more appeals.

5.1.3.2.1 Appeal to Conceptual Knowledge

The appeal to conceptual knowledge resolves an unknowing by the implicit or explicit understanding of the principles governing a domain of mathematics (cf. RittleJohnson & Alibali, 1999). Episode 1 (see Extract 5.13 above) exemplifies this resolution of an Episode.

In line 1, Pearl, a student participant, identifies her unknowing. The question that the teacher participants Jevonte and Deepz had placed on the board was:

$$2x^2 + x - 21 = 0$$

In the knowledge-building part of the Episode (lines 2-8), Teesh explicated Pearl's unknowing, making it accessible to the other participants. Subsequently, in line 9 Jevonte resolved the unknowing by resorting to the conceptual understanding of a principle of algebra, according to which, when a variable has a coefficient of one, the digit "1" is not written. Explicating this knowledge was how Jevonte resolved Pearl's unknowing in line 9, and allowing her to factorise the quadratic equation. This resolution brought the Episode to an end.

An appeal to conceptual knowledge also resolved Episode 9 (see Extract 4.2), where the implicit understanding of the domain of inequalities and the explicit understanding (see Figure 5.2 below) of mnemonic device resolved the Assumed unknowing that triggered the Intention.

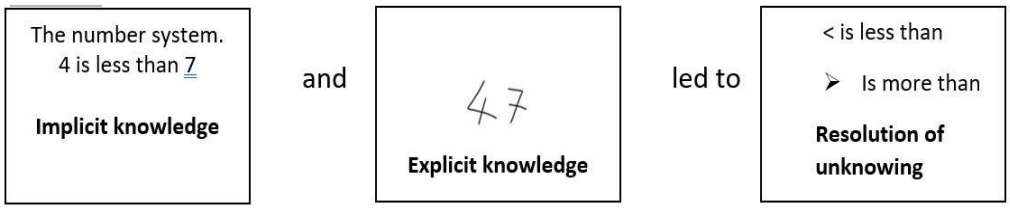


Figure 5.2 – Appeal to conceptual knowledge in Episode 9

5.1.3.2.2 Appeal to a Knower

The appeal to a knower resolves an unknowing by means of the mathematics knowledge possessed by a particular participant. This has the coincidental effect of validating the participant as knowledgeable (cf. Wagner & Herbel-Eisenmann, 2014). The second paragraph of section 5.1.3.1 (acknowledging the resolution of an Episode), in which the resolution of Episode 6 is described, exemplifies how New Knowledge is built through an appeal to a knower. The Episode's Intention, shown in Extract 5.12 above, was triggered by Teesh's Assumed unknowing. She assumed that some participants did not have the mathematics knowledge required to solve quadratic equations by using the quadratic formula. Teesh, the teacher participant, asked Crimson, a student participant, to provide the step-by-step process he used to solve the quadratic equations using the quadratic formulas to all participants.

Extract 5.16 shows Teesh writing Crimson's solution on the whiteboard, and demonstrates her appeal to Crimson as a knower.

Extract 5.16 – Appeal to a Knower – Episode 6

Context: Teesh showed how to use the quadratic formula to solve the question she had posed to the class. Crimson, a student participant, calls out the answer, and Teesh, the teacher participant writes it on the board. Student participants were engaged in comparing their work to what Teesh was writing on the board.

Part	Line	Participant	Action/ Reification	Code
Knowledge Building	10	Crimson (SP):	"Plus, and then the square root" (gestures square root in the air).	Exp-Tells
	11	Teesh (TP):	"Yeah."	Xpt-Controls
	12	Crimson (SP):	"Five squared."	Exp-Tells
	13	Teesh (TP):	"Yeah."	Xpt-Controls
	14	Crimson (SP):	"Minus four."	Expertise-Tells
	(The epistemic interaction continues, with Teesh acknowledging each step.)			

	15	Crimson (SP):	“Over two times six.”	Exp-Tells
	16	Student A (SP):	“Twelve, basically.”	Exp-Clarifies

13:59	17	Teesh (TP):	“And what did you get?”	Xpt-Checks
	18	Crimson (SP):	“Err, zero-point-two-nine-five-threethree-three.”	Exp-Tells
	19	Student A (SP):	“He’s chatting rubbish!”	MR
	20	Crimson (SP):	“This is” (inaudible).	
	21	Jayzee (SP):	“To two decimal places.”	
14:07	22	Crimson (SP):	“Miss said write the whole thing then do two decimal places.”	Exp-Clarifies
14:39	See section 2.1.4			
15:35	23	Crimson (SP) (to Student A):	“You did two minuses, Student A.”	Exp-Clarifies
15:36	24	Crimson (SP):	“It’s not minus five, its ordinary five” (checking her calculation).	

15:53	25	Crimson (SP);	“Yes, you got zero-point-three.”	Exp-Tells
	26	Crimson (SP) (publicly to Teesh):	‘Yes, Student A got it.’	

	27	Teesh (TP):	“Can someone tell me what they got for the minus one?”	Xpt-Controls
	28	Crimson (SP):	“The minus one is one point …”	Exp-Tells
	29	Daniel (SP):	“Wait, slow down, slow down.”	Ext-Requests
	30	Teesh (TP):	“Tell me, how did you put it.”	Xpt-Controls
	31	Crimson (SP):	“In the same way.”	
	32	Teesh (TP):	“Tell me, then.”	Xpt-Controls
	33	Crimson (SP):	(sighs) “Minus five, minus five squared …”	Exp-Tells
	34	Teesh (TP):	“Yeah”	Xpt-Controls
		(The epistemic interaction continues.)		
16:56	35	Teesh (TP):	“So, what did you get?”	Xpt-Controls
17:29	36	Crimson (SP):	“Minus one-point-one-two-six-six-nineseven-nine.”	Exp-Tells
	37	Teesh (TP):	“Who else got this?”	Xpt-Checks

		38	Student A (SP):	"Wait, Teesh, wait, Teesh, let me clarify what Crimson wrote" (as she works out on her calculator).	Ext-Requests
		39	Pearl (SP):	"Jayzee, why, what was your problem?"	Xpt-Checks
		40		(Jayzee discusses with Crimson.)	
		41	Pearl (SP):	(Walking towards Jayzee from where she was seated at the teacher's desk) "Let me tell you what the problem is" (she holds out her calculator and explains the problem to Jayzee).	Xpt-Checks
18:30 New Knowledge		42	Pearl (SP):	(Pointing to the new slide on the board) "Try these ones."	Xpt-Controls
		43	Teesh (TP):	"Try these ones."	

In Extract 5.16, Crimson's epistemic authority resolved Teesh's Assumed unknowing that triggered the Episode. Lines 10 to 36 show the knowledge-building epistemic interactions between participants, in which Crimson has epistemic authority. The step-by-step explanation of his process, which was written on the whiteboard for all participants to see by Teesh, the teacher participant with process authority, resolved the Assumed unknowing in which Teesh was suspended, facilitating the learning of the other classroom participants in their solving of quadratic equations using the quadratic formula.

5.1.3.2.3 Appeal to Procedural Knowledge

The appeal to procedural knowledge resolves an unknowing by executing action sequences for solving mathematics problems (cf. Rittle-Johnson, 2017; RittleJohnson & Alibali, 1999). This resolution of an Intention is exemplified in Episode 2; see Extract 5.17 below.

Extract 5.17 – Appeal to Procedural Knowledge – Episode 2

Context: The lesson started with the teacher participants requiring the student participants to factorise the equation $2x^2 + x - 21 = 0$. This Episode was initiated by Crimson, a student participant. The episode's Intention was triggered by his Identified unknowing of the method for factorising quadratic equations. His dialogical interaction in line 1 oriented the Intention towards Extension. Seeking confirmation from Jevonte, the teacher participant, of the procedure for factorising quadratic equations is evidence of Crimson's uncertainty.

Part	Line	Participant	Action/Reification	Code
Intention	1	Crimson (SP):	"Jevonte, Jevonte."	I
	2	Jevonte:	"Yeah?"	
	3	Crimson (SP):	"So, it has to add to make one and times to make minus forty-two?"	I(Ext) Seeks affirmation

	4	Jevonte:	“Yeah.”	Exp-Affirms
4:09	5	Crimson (SP):	“Seven and minus six … “	Exp-Tells
	6	Teesh (SP):	“Smart. It is!”	MR
	7	(Incoherent chat, with many voices agreeing and giving their solutions.)		
4:30	8	Crimson (SP):	“It has to add to make minus fortytwo, so it will be minus six.”	Exp-Clarifies
	9	Pearl (SP):	“Not add.”	Ext-Challenges
	10	Crimson (SP):	“It’s minus six, minus six plus one equals minus six.”	Exp-Clarifies
	11	Pearl (SP):	“What!”	Ext-Challenges
	12	Crimson (SP):	“Add minus six plus seven.”	Exp-Clarifies
	13	Pearl (SP):	“Minus six plus seven doesn’t give you forty-two, though!”	Ext-Challenges
	14	Crimson (SP):	“Minus six times seven gives you minus forty-two.”	Ext-Clarifies
	15		(More chatter.)	
5:04	16	Teesh (SP):	“Crimson, tell me what you said.”	Ext-Requests
New Knowledge	17	Pearl (SP):	“What?”	
	18	Crimson (SP):	“Minus six times positive seven makes minus forty-two.”	Exp-Clarifies
	19	Pearl (SP):	“Is that not what I said?”	

	20	Deepz (TP):	“Crimson, Crimson, it’s correct.”	

This extract depicts the resolution of an unknowing by an appeal to procedural knowledge. Crimson sought to extend his existing knowledge in line 3 by asking the epistemic question, “So, it has to add to make one and times to make minus fortytwo?” The question he asked concerned the “how” of factorisation: the order of the product and the sum in relation to the coefficients of a, b, and c in the quadratic equation $ax^2 + bx + c = 0$.

In the ensuing knowledge-building interaction, the students had to use their knowledge of multiplication, addition, manipulation of negative numbers, and factors to find two numbers that multiplied to produce -42 and add to produce +1. The discovery of the two numbers satisfied the procedure, thus resolving the Intention and ending the Episode; it was correctly applying a procedure to arrive at the solution that constituted the New Knowledge.

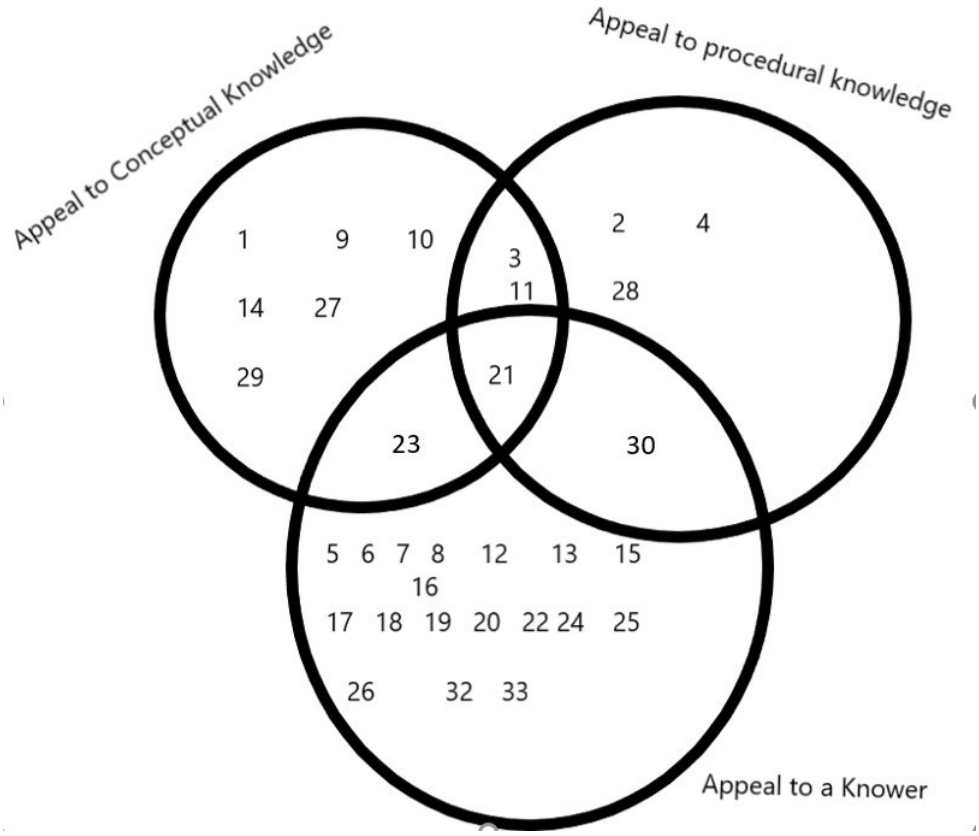


Figure 5.3. – New Knowledge – The dimensions of appeal in all Episodes

I have illustrated each of the three types of appeal; however, Figure 5.3 that shows that in some Episodes there are a combination of two or more types of appeal. In the next section I will illustrate this combination of appeals with a single Episode.

5.1.3.2.4 Appeal to a Knower and Procedural Knowledge

As previously stated, New Knowledge can result from a combination of appeals during the knowledge-building part of the Episode, as is illustrated in Extract 5.18 from Episode 30 below.

Extract 5.18 – Appeal to a Knower and Procedural Knowledge – Episode 30

Context: This is the second lesson on Bounds. Jevonte, a student participant, had volunteered to work out question 3a on the board. As he was writing on the board, Crimson initiated the Episode. His Intention was triggered by an unknowing he had

identified on the board. His dialogical interaction and physical interaction of pointing oriented the intention towards Expertise.

Part		Line	Participant	Action/Reification	Code
Intention		1	Crimson (SP):	(Pointing to board) “He’s doing it wrong” (Identified unknowing).	I (Xpt) Checks
		2	Deepz: (TP)	“Who, who?”	
		3	Student A (SP):	“Apparently, you’re wrong Jevonte!”	Xpt-Checks
	Knowledge Building	4	Jevonte (SP):	(turns from the board) “Who?”	
		5	Student A (SP):	“Apparently, you’re wrong, duh.”	Xpt
		6	Jevonte (SP):	Jevonte: “How …?”	Ext-Questions
		7	Deepz (TP):	“Listen, let him have …”	Xpt-Controls
		8	Jevonte (SP):	“I literally just wrote out the box yeah, what do you expect me to do?”	
		9	Student A (SP):	“Yeah, apparently, its wrong.”	Xpt
		10	Jevonte (SP):	“How is it wrong?”	Ext-Questions

New Knowledge		11	Crimson (SP):	“It’s not zero-point-five, it’s two-pointfive. It is to the nearest five metres; you have to do five divided by two which is two-point-five. Fifty-three is right.”	Exp-Clarifies
		12	Pearl (SP):	“Read the question, read the question.”	Xpt-Controls
		13	Deepz (TP):	(Goes to the board and speaks to Jevonte then turns to Crimson) “You’re right, you’re right.”	Xpt-Checks
		(Interaction continues.)			
		14	Deepz (TP):	“Everyone makes mistakes, that’s why we’re here.”	MR-Motivation

		15	Daniel (SP):	“Why is it one-hundred-and-two-pointfive?	Ext-Questions
		16	Jevonte (SP):	“Ask him,” (points to Crimson) “that’s what he told me.”	
		17	Daniel (SP):	“Why are you listening to him?”	
		18	Deepz (TP):	“It’s right.”	Xpt-Checks
		19	Crimson (SP):	“It’s right, it’s to the nearest five metres, so you divide by two.”	Exp-Clarifies

		20	Deepz (TP) (to Jevonte at the board):	"Right, to the nearest five metres you do five divided by two, so the lower bound is one-hundred-and-two-point-five."	Xpt-Controls
		21	Pear (SP):	"Wait ... Shouldn't it be one-hundred-and-three-point-five?"	Ext-Questions
		22	Deepz (TP):	"No, you are taking away two-point-five."	Exp-Clarifies
		(Discussion continues.)			
		23	Deepz (TP):	"Who doesn't understand It?"	Xpt-Checks
		24		(Inaudible response.)	
		25	Deepz (TP);	"What bit don't you understand? Do you understand why we did five divided by two? Do you understand that?"	Xpt-Checks
		26		(Inaudible response.)	
	New Knowledge	27	Deepz (TP) (to the class):	"Say if we are trying to the nearest 8 metres what will we do?"	Xpt-Checks
		28	Pearl (SP):	"Twelve, you add and subtract four."	Exp-Tells
			Deepz (TP):	"Yes, Who has done 3b?"	

In Extract 5.18, the New Knowledge was realised by Crimson in line 11. Crimson, as an epistemic authority, Clarifies the relevance of the phrase 'the nearest five metres'. This New Knowledge was the result of an appeal to Crimson as a knower, and had

the potential to resolve an unknowing. However, the Episode did not end, as not all participants as yet acknowledged the resolution of Crimson's Identified unknowing; knowledge building was still underway for the rest of the class. Further acknowledgement was as a result of an appeal to procedural knowledge, that is, to the adding and subtracting after dividing by 2 as illustrated in line 19. In line 20, Deepz, the teacher participant, asked Jevonte to write it on the board for all participants to copy down. It was this appeal to procedural knowledge that allowed all participants to acknowledge the resolution of the unknowing bring about the end of the Episode. The New Knowledge was, therefore, ultimately the result both of Crimson Explicating his mathematics knowledge as a knower and Deepz appealing to the procedural knowledge of adding and subtracting following a division by 2, both of which together allowed the other participants to acknowledge the resolution of the unknowing.

5.2 Elaborating on Participants' Interactions

In this section, I elaborate on the findings that emerged from the interactions between participants in the mathematics classroom as they directed their agency towards the learning of mathematics. As stated in the introduction, this research addresses the emergence of shared epistemic agency amongst the participants of my classroom as they enacted my innovative pedagogy. The framework established through my literature review suggested that shared epistemic agency was encapsulated by the six characteristics summarised in section 2.4.3. However, I was particularly interested in the interactions of these characteristics as participants enacted the innovative pedagogy, and how the interactions highlighted what was unique about the participation in my classroom as an indicator of how the research

was meeting the aims of the study. To do this, I focused on three features of the pedagogy indicated in the literature (see section 2.3.2), considering the positioning of participants during interaction, how process authority manifested in the classroom, and how participants' epistemic authority impacted on the way mathematics knowledge was advanced in the classroom.

5.2.1 Positioning

As participants interacted in the classroom for the purpose of learning mathematics, they assigned positions to themselves and to other participants in the process. Positioning can be descried as the discursive process by which speech and action are used to arrange people in social structures through locating them in conversations as participants in jointly-produced ongoing repertoires that are elements of a shared culture, or which can be invented as participants interact (see section 2.3.2.2). Positioning someone establishes what others must do for them or what they must do for others. Positions differ from roles; while positions are contextspecific and flexible, as participants can occupy more than one position and shift between positions, roles are static in their interactions.

The roles of student participant and teacher participant have been hitherto used to distinguish between the functions of participants in each Episode as they enact the innovative pedagogy. However, as shown in the extracts used to elaborate upon the various modes, the knowledge-building characteristics of shared epistemic agency are not specific to these roles. Extension is not restricted to student participants; neither is Explication nor Expertise restricted to teacher participants, as one would expect when considering the conventional relationship between students and

teachers. As exemplified in Extract 5.19 below and as is typical in other Episodes, enacting the innovative pedagogy made available to participants the positions of the learner, knower, and facilitator.

The position of learner is associated with Extension; the position of knower is associated with Explication; and the position of facilitator is associated with Expertise. I borrow the term “facilitator” from Kolb et al. (2014), and use it in the sense of actively supporting learning – that is, creating conditions that enable others to learn and removing obstacles that prevent others from learning (p. 7). The difference between a position and the associated characteristic of shared epistemic agency is that while a participant directs their agency toward one of these characteristics, the position is more than the actions and reifications of participants, and is also about how a participant is viewed by others during interaction, what others must do for them, and/or what they must do for others whilst in these positions. While a participant can direct their agency towards explicating their mathematics knowledge, and their actions or reifications will be coded as Explications, the positioning can occur prior to as well as during these a participant’s actions and reifications, as they can be positioned by another participant before they begin speaking.

The findings demonstrate that a participant can be positioned interactionally, in a given moment, by others as a knower or as a facilitator. They can be positioned reflexively, in a given moment, by themselves as a learner, knower, or facilitator, and can be positioned institutionally by the pedagogy as a facilitator when ascribed the role of teacher participant. The interactions in Episode 2 best demonstrate these positions, as illustrated in Extract 5.19 below.

Extract 5.19 – Positioning – Episode 2

Context: The teacher participants, Jevonte and Deepz, positioned institutionally by the pedagogy as facilitators, had asked the student participants to factorise the equation $2x^2 + x - 21 = 0$.

Part	Line	Participant	Action/Reification	Code	Positioning
Intention	1	Crimson (SP):	"Jevonte, Jevonte" (Identified unknowing).	I (Ext)	Crimson positions himself as a learner. Crimson positions Jevonte as a facilitator. Jevonte has been positioned as a facilitator by the pedagogy.
	2	Jevonte (TP):	"Yeah"		Jevonte accepts the position of a facilitator.
	3	Crimson (SP):	"So, it has to add to make one and times to make minus forty-two?"	I (Ext) Seeks Affirmation	Crimson positions himself as a learner. Crimson positions Jevonte as a knower.
	4	Jevonte (TP):	"Yeah"	Exp Affirms	Jevonte accepts the position of a knower.
4:09	5	Crimson (SP):	"seven and minus six …. "		Crimson positions himself as a knower.

	6	Teesh (SP):	“Smart. it is!”	MR Non contextual conducive	Teesh positions Crimson as a knower.

	7	(Incoherent chat, many voices agreeing and giving their solutions.)			
4:30	8	Crimson (SP):	“It has to add to make minus forty-two, so it will be minus six.”	Exp-Clarifies	Crimson positions himself as a knower.
	9	Pearl (SP):	“Not add?”	Ext-Challenges	Pearl positions herself as a learner.
	10	Crimson (SP):	“It’s minus six, minus six plus one equals minus six.”	Exp-Clarifies	Crimson positions himself as knower.
	11	Pearl (SP):	“What!”	Ext-Challenges	Pearl positions herself as a learner.
	12	Crimson (SP):	“Add minus six plus seven.”	Exp-Clarifies	Crimson positions himself as a knower.
	13	Pearl (SP):	“Minus six plus seven doesn’t give you forty-two, though!”	Ext-Challenges	Pearl positions herself as a learner.

	14	Crimson (SP):	“Minus six times seven gives you minus forty-two.”	Exp-Clarifies	Crimson positions himself as a knower.
	15		(More chatter.)		

5:04	16	Teesh (SP):	“Crimson, tell me what you said.”	Ext-Requests	Teesh positions herself as a learner, and Crimson as a knower.
New Knowledge	17	Pearl (SP):	“What?”		
	18	Crimson (SP):	“Minus six times positive seven makes minus forty-two.”	Exp-Clarifies	Crimson positions himself as a knower.
	19	Pearl (SP):	“Is that not what I said?”		
	20	Deepz (TP):	“Crimson, Crimson, it’s correct.”	Xpt-Checks	Deepz positions himself as a facilitator. Deepz confirms Crimson in his position as a knower.

Extract 5.19 shows that positions are flexible as a participant can be positioned or position themselves as either a learner, knower, or facilitator from moment to moment. Crimson initiated the Episode as a learner, and by line 5 has positioned himself as a knower. In line 1, Crimson positions Jevonte as a facilitator, and Jevonte accepts this positioning in line 2. By seeking affirmation from Jevonte in line 3, Crimson positions him as a knower with epistemic authority. Having been positioned as a knower by Crimson, Jevonte accepts the position in line 4 and Affirms the latter's knowledge. This demonstrates how one participant can position another. In line 5, Crimson also starts to position himself as a knower, and in line 6, Teesh acknowledges this positioning. By line 16, Teesh, in seeking to extend her knowledge, positions Crimson as a knower; Deepz further confirms this positioning in line 20.

This interplay of positioning extends the possibility for describing participants as student participant learners (SL), student participant knowers (SK), student participant facilitators (SF), teacher participant learners (TL), teacher participant knowers (TK), or teacher participant facilitators (TF) in the context of a given situation within an Episode. Although I am referring to a participant who is learning, knowing, or facilitating, I am not reducing knowledge building to the individual experiences associated with these positions; rather, I am emphasising the publiclyrecognised capacities and criteria for being a learner, knower, and facilitator. In this way, one's positioning is constituted by the community, or sections therein.

5.2.1.1 Positioning as a Learner

Enacting the innovative pedagogy, the participants positioned themselves as learners by seeking knowledge from an external source such as another participant, as shown in Extract 5.19 above. While, in theory, a participant can, for instance, be positioned as a learner by another participant, in the discourse of conventional pedagogy, a student is institutionally positioned by the teacher as a learner in the classroom by default. This positioning did not emerge in the practice developed in this study. Instead, while a participant could themselves assume the position of learner, a participant could not position another participant as a learner. A participant positioned as a knower in an epistemic interaction does not automatically confer the position of learner upon the other participants. The learner has to position themselves.

This reflexive positioning as a learner could show that in an epistemic interaction, being a learner or making the decision to learn in order to extend one's knowledge is a decision an individual makes for themselves in the moment. This difference in positioning, when compared with the situation in a conventional pedagogy, will be discussed further in chapter 6.

5.2.1.2 Positioning as a Knower

My analysis shows that positioning as a knower is based on who claims to be knowledgeable, or who represents themselves as having epistemic authority in a given moment. The data shows that if participant A is seeking knowledge from

participant B, it can be taken to show that participant A sees participant B as knowledgeable, and therefore as a knower. This positioning contrasts with a further scenario in which participant A decides to share their knowledge with participant B; in this scenario, participant A sees themselves as knowledgeable, and positions themselves in their interactions with participant B as a knower.

5.2.1.3 Positioning as a Facilitator

The analysis of the video recordings showed that the participants of my classroom directed their agency towards expressing process authority (see section 5.2.3), and, in this way, facilitated the advancement of mathematics knowledge in the classroom community. The modes of Expertise – Controls, Checks, Manages (see section 5.1.2.3) – suggest the ways participants control the learning behaviours of participants in the classroom (see section 5.1.2.3.1), check their current knowledge (see section 5.1.2.3.2), and manage the learning resources (see section 5.1.2.3.3). These actions and reifications exemplify the participant as a facilitator of mathematics learning; by directing their agency towards expressing process authority, they facilitate the "how" of the learning in the classroom community.

As can be seen in Appendix 8, which presents the tally of the attestations of knowledge-building characterisations from my review of the video recordings, teacher participants were positioned or positioned themselves as facilitators twice as many times as student participants. This difference is evidently the result of the pedagogical measures that require the participants to take on the role of teacher participants, urging them to assume process authority while in this role. In the role of

teacher participants, they are expected by the pedagogy (see section 3.1) to plan the structure of the lesson structure, and to take responsibility for how knowledge is made explicit and communicated to other participants so that their mathematics knowledge is advanced. Having been positioned by the pedagogy as a facilitator, teacher participants were also interactionally positioned as facilitators by other participants within the lesson, while, on occasion, student participants also reflexively positioned themselves as facilitators. There was no evidence of a student participant being positioned as a facilitator by another participant. A logical explanation could be that participants associated the teacher participants with the conventional role of the teacher, and expected them, while they occupied the role, to take responsibility for the learning culture in the classroom, but did not expect this of other student participants. Thus, there were three observed ways in which positioning as a facilitator can occur: institutionally (teacher participants, by the pedagogy), interactionally (teacher participants, by other participants), and reflexively (student participants, by themselves).

5.2.1.4 The Learner and the Knower as Productive Agents

An Episode of shared epistemic agency is productive, as the outcome is New Knowledge (section 4.1.3). The knowledge-building phase is that in which the Intention to resolve an unknowing through epistemic interaction leads to the production of this New Knowledge. The findings show that the positionings of learner and knower iteratively and reciprocally constitute each other during this phase, as exemplified in Extract 5.20 below.

Extract 5.20 – Learner/Knower as Productive Agent – Episode 23

Context: This is the second lesson on composite functions. The previous lesson ended with the working out of solutions to the booklet questions on the board. At the start of this lesson, as a continuation of the previous day's lesson, teacher participant James called on Crimson, a student participant, to come to the board and show the class how to solve a question on composite functions.

Part	Line	Participant	Action/Reification	Code	Positioning
Knowledge Building	1 1:52	Crimson (SP):	"…the answer you get from 'g,' you put into 'f,' you understand …"	Exp-Clarifies	Crimson positions himself as a knower.
	2		(Murmurs of acknowledgment from participants.)		Participants positions Crimson as a knower.
	3	Pearl (SP):	"Couldn't you start with ern …"	Ext-Seeks affirmation	Pearl positions herself as a learner. Pearl opens up the position of knower.
	4	Student A (SP):	"… 'gh'?"	Exp-Explicates unknowing	Student A positions herself as a knower.
	5	Daniel (SP):	"No."	Exp-Affirms	Daniel positions himself as a knower.
	6	Crimson (SP):	"Yes, you could if 'f' wasn't there …"	Exp-Affirms & Clarifies	Crimson positioning self as knower.
	7	Pearl (SP):	"Yes, in front of 'h'…"	Exp-Articulates knowledge	Pearl positions herself as a knower.

	8	Student A (SP):	“Couldn’t you start with ‘fh’?”	Ext-Seeks Affirmation	Student A positions himself as a learner. and positions Crimson as a knower.
	9	Crimson (SP):	“You could do five squared minus equals twenty-five.”	Exp-Affirms & Clarifies	Crimson positions himself as a knower.
	10	Pearl (SP):	“5 squared? Why you started with 5 squared ...?”	Ext-Questions	Pearl positions herself as a learner.
	11	Student A (SP):	“‘Cause you put it into the …”	Exp-Clarifies	Student A positions herself as a knower.
	12	Pearl (SP):	“Yeah, yeah …”	Exp-Clarifies	Pearl positions herself as knower.

Pearl positioning herself as a learner in line 3 prepared the opportunity for other participant to position themselves (and be positioned by her) as knowers in lines 4-7. Furthermore, it led to Student A positioning herself as a learner in line 8. Each line in the interaction acted to produce the next as part of the knowledge-building interaction. Crimson (SK), in line 1, shares his knowledge of composite functions with the other participants. This sharing causes Pearl, in line 3, as a learner, to seek affirmation. In this moment, Student A, as a knower, acts to explicate Pearl's unknowing, and by this dialogical interaction, helps other participants to make sense of Pearl's Extension. This exchange leads to Daniel positioning himself as a knower, but he does not follow through with his Explication, and this opens up the opportunity for Crimson to position himself as a knower. Positioning herself as a knower, having been the learner who started off the interaction, Pearl finishes off Crimson's Explication in line 6. In this way, Pearl was able assume an active role in her own knowledge advancement and that of her peers.

The productive quality of the interaction is referred to as "productive agency" (Schwartz & Okita, 2004), in that, when Pearl sought to extend her knowledge in line 3, she did not know what impact it would have, but Crimson, Student A, Daniel, and herself actively (that is, as agents) built on each other's knowledge and modified it, ultimately producing New Knowledge. This productive interaction is the "sharedness" of shared epistemic agency that advances the knowledge of the classroom community.

The research questions are concerned with the characteristics of shared epistemic agency as it emerges, and positioning during epistemic interaction is a significant example of these. The characteristics are more complex during the interaction than is suggested by their definitions alone; Extension does not simply signify a lack of knowledge, and could be a form of authority; moreover, the corresponding positions have a productive impact on each other. I will draw on these qualities of the positions in the discussion chapter.

5.2.2 Process Authority in Interaction

The process dimension of authority refers to who is in control of the culture of learning in the classroom – that is, of how the learning takes place (see section 2.3.2). Building on my analysis of process authority through participants' interactions, I highlight three issues: a blending of authority amongst participants as they enacted the innovative pedagogy; a freedom to pursue dialogical and physical interactions in the classroom; and the emergence of the learner as having implicit control over the other participants' behaviour.

5.2.2.1 Blending of Process Authority

As facilitators, participants did not take on all the responsibilities associated with the conventional teacher; in facilitating the advancement of knowledge, they blended their authority with mine. Enacting the innovative pedagogy made clear that advancing the knowledge of other participants in a secondary mathematics

classroom community required more than the subject content knowledge that constitutes epistemic authority. The teacher participants did not have the mathematics knowledge for teaching (see section 2.3.2.1) that accompanies the possession epistemic authority. As stated in chapter 2, process authority in this study subsumes pedagogic content knowledge and curricular knowledge that is conceptualised as mathematics knowledge for teaching. Mathematics knowledge for teaching includes knowledge of the scope of the mathematics topic to be taught in a given lesson, the prerequisite understanding required to engage with the topic, and the relationship between the topic and the examination requirements.

In teaching cycle 2, teacher participants Deepz and James were required to teach the topic “speed, distance, and time”. Their primary source in their preparatory research was the MathsWatch virtual learning environment (VLE) to which the school subscribes. Hence, they focused their lesson on the time-distance graphs that they encountered on the platform; they did not extend the topic to questions on speed, distance, and time calculations, which were the more typical foci of examination questions. They did not have the additional knowledge of exam requirements, nor of the scope of the topic.

On the other hand, teacher participants Adam and James, who taught inequalities in teaching cycle 3, extended their discussion of the mathematics topic to solve linear inequalities and quadratic inequalities using sketches of quadratic graphs. They did not have the mathematics knowledge for teaching that causes a teacher to structure an instructional sequence in terms that are intelligible to the learners by laying the foundations for learning other ideas. This knowledge would have positioned the

drawing of quadratic graphs as a prerequisite for solving quadratic inequalities, and assimilated this technique into a network of ideas that are important to students' reasoning.

I remedied this situation by bringing my mathematics knowledge for teaching to enable the participants to advance the community's mathematics knowledge. In this way, there was a blending of process authority between the participants and myself. I contributed my mathematics knowledge for teaching to support the participants' enactment of the innovative pedagogy without usurping their authority. From teaching cycle 3 onwards, I produced a booklet of mathematics questions for each teacher participant, which became our reference material. This booklet, a reification of the appropriate mathematics knowledge corresponding to each mathematics topic, equipped the teacher and student participants with a representation of the boundaries of the relevant mathematical knowledge. The possession of the booklet placed the teacher participants in the same position as conventional mathematics teachers who use a textbook. The teacher participants could focus on advancing community knowledge rather than on preparing resources, which became my primary role. Producing the booklet evidences how the participants and I negotiated the blending of process authority over time; I produced the resource while they themselves managed this and other resources.

The emergent blending of process authority led to a change in my role on a lessonby-lesson basis, dependent on the procedures and requirements of the teacher participant. Institutionally positioned as facilitators by the pedagogy, the teacher participants directed whether I was to take on the role of teaching assistant

or student participant in the lesson, asserting their authority by renegotiating my authority, while, at the same time, calling upon my authority as the teacher when they chose to do so. For instance, in Extract 5.15 (see section 5.1.2.3.3), in line 1, in my role as the teacher, I asked Crimson about the cause of the delay in starting the lesson; he responded by stating that there was no reason for the delay (positioning himself as a facilitator). In line 7, he announced to the class that I would hand out the booklets (positioning me as a facilitator), though I reminded him that the students already had booklets (see line 8). As Deepz did not have a booklet, I went into the office and got him a spare booklet (positioning myself as a facilitator). Upon my return to the classroom, Crimson asked me to sit down and do the work, saying, "Miss could you sit down please?" (positioning himself as a facilitator by repositioning me as a student). I handed the booklet to Deepz, reminded Ty to focus on his work (asserting my facilitator position), and sat down to act as a student (accepting Crimson's positioning of me as a facilitator).

5.2.2.2 Control of Social Behaviour (Freedom of Dialogical and Physical Interaction)

Participants took individual control of their dialogical and physical interactions in the classroom. Positioned institutionally as facilitators, teacher participants often controlled the epistemic behaviours of other participants. However, individual participants also took control of their own social behaviours as they sought to advance their mathematics knowledge. As epistemic interactions occurred from

moment to moment in the classroom, the spontaneous, liberal performance of physical interactions around the classroom and dialogical interactions with other participants became a central aspect of the classroom practice. Attending to these interactions reveals that participants physically moved around the classroom to interact with other participants; without restriction, they entered into or initiated dialogical interactions with other participants as they saw fit, in order to advance their individual knowledge and that of other participants. Extract 5.21 shows how participants engaged in epistemic interactions without restriction.

Extract 5.21 – Control of Social Behaviour – Episode 10

Context: The teacher participant James explicates knowledge to the classroom community concerning the representation of inequalities on a number line. Student participants were focused on him and his Explication. The Episode was initiated by James, the teacher participant, whose Intention was triggered by an Assumed unknowing. He assumed that a lack of knowledge of inequalities existed amongst the classroom participants. As such, his Intention was oriented towards Explication. Adam, the other teacher participant, was at the teacher's computer, managing the learning resource – the PowerPoint lesson plan.

Part	Line	Participant	Action/Reification	Code	Positioning	Movement/ Communication
Intention	1	James (TP):	"If you want to plot this here, so we know that its less than, so we put a circle …" (Assumed unknowing).	I (Exp)-Clarifies	James positions himself as a facilitator	
Knowledge Building	2	Student B (SP):	"… and you colour it in, right?"	Ext-Seeks affirmation	Student B positions herself as a learner	Student B calls out from seating position
	3	James (TP):	"… yeah, you colour in the circles because its less than …"	Exp-Affirms then Clarifies	James positions himself as a knower	
	4	Crimson (SP):	"… And then you draw an arrow down…"	Exp-Clarifies	Crimson positions himself as a knower	Crimson calls out from seating position

	5	James (TP):	“Then you draw an arrow down.”	Exp-Clarifies	James positions himself as a knower	
	6	Student A (SP):	“Wait, wait, I got a question!”		Student B positions herself as a learner and opens up the position of knower	Student A calls out from seating position
	7	James (TP):	“Yes?”		James positions himself as a facilitator	
	8	Student A (SP):	“So, if its more than you draw an arrow that,” (pointing towards her right) “across the way?”	Ext-Seeks affirmation	Student A positions herself as a learner	Student A calls out from seating position
	9	James (TP):	“Yes.”	Exp-Affirms	James accepts the position of knower,	

					and positions himself as such	
1:19	10	Crimson (SP):	“But if it’s not equals to, don’t colour it. You see, when it says equals to you, colour in the dot. If it does not say equals to, you don’t colour in the dot.”	Exp-Clarifies	Crimson accepts the position of knower by positioning himself as such	Crimson talks across to student A seated two positions away
		Student B (SP):	“I get that.”			
1:19	11	Student B (SP):	“Can I do the question on the board, please?”	Ext-Requests	Student B positions herself as a learner, opening up the position of facilitator	Student B stands up and comes to the board; James holds out the pen to her

	12	James (TP):	“Yes, sure, do you know how to do it?”	Xpt-Controls & Checks	James positions himself as a facilitator	
1:30	13	Daniel (SP):	“Do you have to draw a number line?”	Ext-Seeks Affirmation	Daniel positions himself as a learner	Daniel calls out from the seat
1:39	14		(Jevonte stands up, walks over to communicate with a participant, and walks back.)			

The last column of this extract exemplifies the learning activities that became typical of the classroom community. Participants communicated with each other when they saw fit to do so. In line 2, Student B, by positioning herself as a learner, contributes to the Explication by interjecting with the question "…and you colour it in, right?" Though ostensibly a question with which she is seeking affirmation of her knowledge (Extension), this phrase also contributed to community knowledge as it was asked and responded to publicly. Crimson, in line 4, continues James's Explication with "and then you draw an arrow down." James repeats his exact phrase in line 5, while correctly drawing the line to the left. He thus appears to understand that by "down", Crimson actually meant "to the left". So, by drawing the line to the left, he legitimised Crimson's contribution. Student A contributed to the Explication with her presentation of a question in lines 6 and 8. Her question further legitimised the term "down" as meaning "to the left". In line 10, Crimson further contributes to knowledge advancement by Explicating knowledge to Student B. In the recording, James becomes inaudible towards the end of his sentence in line 3, and did not finish expressing his thought. Crimson may have felt responsible, as his interjection in line 4 interrupted James; this may be why he decided to repeat himself more clearly in a public statement in line 10, setting the process of collective knowledge advancement that he had threatened to disrupt back on track.

Physical movement also occurred at will in the classroom. In line 11, Student B stands up and walks to the board, wanting to extend her knowledge by publicly working through a solution. James' acceptance of her behaviour is evidenced by his giving the pen to her. In line 14, Jevonte walks across the class, communicates with a student, and then returns to his seat. The reasoning behind this interaction is unclear, but he moved of his own volition and did not distract participants from their learning.

5.2.2.3 The Position of the Learner as Authority

My analysis of the data points to the possibility of considering the learner position as a source of process authority in the classroom. This authority manifested as the ability to cause other participants to behave in specific ways. Analysing participants' actions when positioned as a learner, and their impact on other participants during epistemic interactions, is pertinent to the research questions. Process authority is in the possession of the participant who seeks to extend their existing knowledge by causing other participants to act in ways that allow the participant in question to extend their existing knowledge. This is clear in Extract 5.10, wherein the whole class goes over a question on the board. The teacher participant Teesh asked the student participants what they did at each point of the working out, and she wrote down their responses on the board; some students were checking their work against these answers. In line 1, Student A reflexively positions herself as a learner,

publicly declaring to the class, “I didn’t get that.” This caused the lesson to come to a halt, with all attention devoted to helping Student A investigate where she went wrong. Line 4, where Student B say, “Are you sure, Student A, because it happened last time,” indicates that this act of publicly articulating her unknowing and causing the class to pause had happened on a previous occasion. Student A displayed the same authority in line 38, visible in Extract 5.16 of the same Episode, where she said, “wait Teesh, let me clarify what Crimson wrote”; again, by making a public statement, she halted the pace of the lesson, and having checked her work, Student A indicated that the lesson could continue. Similarly, in line 29, Daniel positions himself as a learner, saying, “wait, slow down, slow down,” causing the teacher participant to pause and not write anything on the board for a few moments so that Daniel could copy what was already written.

Positioning oneself as a learner can position another participant as a knower, thereby requiring the other participant to Explicate their mathematics knowledge. For instance, in Extract 5.19 (see section 5.2.1), in line 3, Crimson, in positioning himself as a learner seeking affirmation, implicitly positions Jevonte as a knower. Crimson self-positioning can be said to have caused Jevonte to act in a certain way – that is, to accept the position of a knower and to Explicate knowledge.

This evidence of the learner position as a source of authority points to the relational agency of the participants (cf. Edwards, 2005). Relational agency is the ability to align one's thoughts with those of other

participants, to recognise what they need to achieve their goals, to interpret other participants' problems, and to respond to this interpretation. This ability marks the classroom environment as a safe space wherein participants are free to share their lack of knowledge, with the trust that the community will do what it takes to help them know.

5.2.3 Epistemic Authority in Interaction

The epistemic dimension of authority refers to who is validated as a knower, i.e., who is viewed as legitimately knowledgeable (see section 2.3.2). In my analysis of epistemic authority in participants' interaction, I highlight three issues: knowledge as a prerequisite for extension; a disregard for ability labels; and the individual and communal responsibility for knowledge advancement.

5.2.3.1 Knowledge as a Prerequisite for Extension

The data points to knowledge as a prerequisite for knowledge-building interaction; this prerequisite is inclusive of Extension, the characteristic of shared epistemic agency that focuses on extending a participant's existing knowledge. For a participant to direct their agency towards Extension requires the possession of certain background knowledge.

Knowledge is required for all modes of Extension, as exemplified in Extract 5.8 (see section 5.1.2.1.4), line 3: "So, it has to add to make one and times to make minus forty-two?" This example of Seeking affirmation as a mode

of Extension is made possible by the presence of some procedural factorisation knowledge, however incomplete. From my personal assessment of the participants at this point in time, I discerned that Crimson had some knowledge of factorising an expression ax^2+bx+c. He knew that the coefficient "a" needed to be multiplied by the constant "c". He also knew that the solution lay in the multiplication and addition of the correct figures. His unknowing, which needed resolution, was whether the product or addition gave the coefficient of "b" or the constant "c". Teesh's assessment of Crimsons' solution as "smart" was based on her personal confirmation that his solution was correct, by checking that expanding the factorisation resulted in the original expression. This acknowledgment also required knowledge of factorisation.

Extract 5.1 (see section 5.1) provides another example of knowledge being a necessity of Extension in knowledge-building interactions. In line 3, Pearl's Extension by means of the epistemic Question, "But what do you times together to get x?" could not have been made if she did not have knowledge of factorising quadratic equations with a coefficient besides the integer 1. It is this knowledge that allows the participant who seeks to extend their existing knowledge to challenge an Explication that does not advance their existing knowledge, and to recognise when their knowledge has been advanced. Extract 5.7 from Episode 3 (see section 5.1.2.1.3) illustrates how Extension in the mode of a Request also requires certain prerequisite knowledge. In this extract, Teesh requests to Extend her knowledge of solving quadratic equations. In lines 3, 5, and 7, she requests permission from the teacher participant to solve the

quadratic equation by doing what she knows and building on it in front of the class; this requires at least a provisional grasp of the associated principles. Summarily, Extension, in all four of its modes, does not indicate a total lack of knowledge, but in fact reveals a comprehension of certain prerequisite principles.

5.2.3.2 Disregarding Presumed Ability Labels

On entry, the school assigned students to ability bands based on their performance in the standardised assessments taken by all students in the UK at the end of their primary school education. If this information is unavailable, the school will assign a band from performance in the school's entry assessments. These bands indicate students' predicted range of attainment at the end of their secondary schooling.

Students could be assigned to any of the 1-2 (foundation), 2-4 (lower), 4-6 (middle), 6-9 (higher), or 7-9 (higher plus) attainment bands. My mathematics class comprised a selection of students from the lower, middle, and higher attainment bands.

Regardless of the band to which they were allocated, all participants enacted the pedagogy as both teacher sand student participants. During the Select stages of the pedagogy, wherein participants selected their mathematics topic to teach (see section 3.1), all topics were available for selection. I did not consider the presumed level of difficulty of the topics or the participants' ability band, nor did participants appear to do so. This lack of consideration shows that the pedagogy and its enactment did not

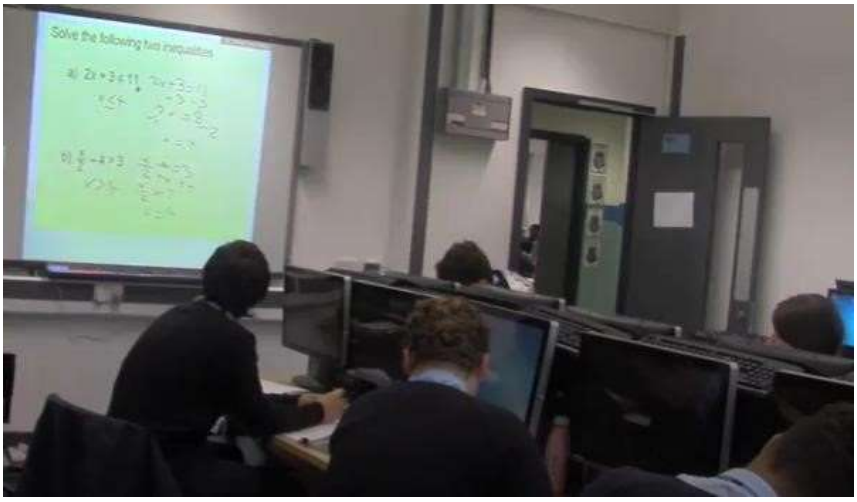
recognise the ability levels of the participants; nor did the participants consider the associated labels in proceeding with their learning.

At the end of each teaching cycle, when I reflected on the pedagogy (see section 3.4.2.3) I found no link between the quality of the enactments and the presumed ability of the participants; hence, I implemented no measures in subsequent selection stages to constrain which participants selected which mathematics topic. It remained open and democratic.

That participants participated in all stages of the pedagogy indiscriminately and collaboratively is evidence of their rejection the ability labels. They all prepared for the lessons, creating knowledge objects in the form of the PowerPoint presentations that reified their mathematics knowledge; they all shared their mathematics knowledge and reflected on each other's performance. Participants' acknowledgment of each other's performance is further evidence of this rejection (see Photograph 5.3).

Photograph 5. 3 – Disregarding Ability Labels – Episode 11

Context: Teacher participants Adam and James take turns to lead a discussion of different aspects of inequalities. Adam demonstrates the method of solving inequalities by going over the solutions for equations that have been taught previously. At the end of Adam's Explication, as shown in Photograph 5.3, the student participants spontaneously start clapping.



Photograph 5. 3 – Disregarding presumed ability labels

When questioned, the participants responded that the clapping showed appreciation for how Adam connected their previous knowledge of solving linear equations with the solving of inequalities. The clapping represented the ease with which they could now advance their mathematics knowledge, and their appreciation of Adam for making this happen. Unbeknownst to them, Adam was in the lower ability band; their enthusiastic appreciation of the mathematics knowledge that he shared shows the disregard, on his part and the part of others, for the mathematics ability labels imposed upon them by the school.

5.2.3.3 Individual and Community Knowledge and Responsibility

Enacting the innovative pedagogy dictated by the pedagogic principles (see section

2.5.1), the participants took responsibility for advancing their mathematics knowledge. The pedagogy prescribed that the teacher participants take responsibility for advancing the knowledge of the student participants

during the Plan and Share stages of each teaching cycle (see section 3.1). However, what was not prescribed by the pedagogy, but which nevertheless emerged and evidenced in the data, was the fact that the community took responsibility for individual participants' knowing and unknowing.

I use Extract 5.22 as an example. This extract is an expanded representation of the classroom chatter indicated between lines 5 and 6 of Extract 5.10 (see section 5.1.2.1.5). This classroom chatter is numbered as 5.1-5.8, and shaded in Extract
5.22 below.

Extract 5.22 – Individual and Community Knowledge and Responsibility – Episode 6

Context: Teesh shows how to use the quadratic formula to solve the question she earlier posed to the class. Crimson calls out the answers, and Teesh writes them on the board. At 14:39, Student A performs a dialogical interaction.

Part	Line	Participant	Action /Reification	Code
Knowledge Building	1	Student A (SP) (to the class):	"I didn't get that, but I got the same calculation in my calculator."	Ext
	2	Jayzee (SP):	"What did you get?"	Xpt

	3	Student A (SP) (to Jayzee):	"I got ..."	
	4	Student B (SP):	"Are you sure, student A, because it happened last time …"	Xpt
	5	Teesh (TP):	"Did everyone get this? … someone got this, yeah?" (waits for responses).	Xpt
	5.1	James (SP):	"Now I got it."	
	5.2	Teesh (TP):	"Eh?"	
	5.3	James (SP):	"I got it now."	
	5.4	Crimson (SP):	"Did you get it?"	
	5.5	Daniel (SP):	"How did you get ..."	
	5.6	Daniel (SP):	"…Oh yes! Squared!"	
	5.7	Crimson (SP):	"Oh yes, Student A …"	
	5.8	Teesh (TP):	"Everyone got this?"	
	6	Student A (SP) (To Teesh):	"I didn't get it."	Ext

	7	Crimson (SP) (turning to Student A with surprise):	“Oh, you didn’t? What did you get?”	Xpt
	8	Student A (SP):	“I put this in my calculator,” (she passes her calculator to Crimson, who studies it).	Ext
	9	Teesh (TP) (to the class):	“So, who got the one with the minus then?”	Xpt
	10	Crimson (SP) (to Student A):	“You did two minuses, Student A.”	Exp
	11	Crimson (SP) (to Student A):	“It is not minus five; it’s ordinary five.”	Exp

Teesh demonstrates her responsibility for the community knowledge by her repeated inquiry in lines 5, 5.8, and 9; she wanted to gauge and make clear what each participant knew and what they did not know. Extract 5.12, the continuation of this Episode, demonstrates her encouraging Crimson to articulate his knowledge in a structured step-by-step way for the advancement of participants’ knowledge.

This extract shows how knowing or not knowing was important to the community. In lines 5.1-5.8, the public demonstration of participants' knowing and unknowing corresponded to a practice that had become standard in the classroom; this practice is evidence that the sharing of knowledge was considered to be valuable by all participants, whether teacher or student.

In line 1, Student A states that she did not get the answer written on the board.

What follows indicates how the community took responsibility for ensuring that Student A's unknowing was alleviated. In the first instance, Teesh, the teacher participant, pauses her explanation of the calculation on the board. Notably, no participant complains either about Student A's interruption or about Teesh pausing her explanation, even though it appears, as line 4 suggests, that Student A had stopped the lesson before for an unjustified reason. In addition to this display of patience, the Explication performed by Teesh and discussed in lines 2, 4, 5.4, 5.7, and 7 represents the willingness to help Student A to know on the part of the other participants.

This extract demonstrates that a participant's knowing and unknowing was considered to be the community's property, and that the community's unknowing and knowing was the individual's responsibility.

5.3 Summary

This chapter detailed the findings of the analysis of the thirty-six Episodes of shared epistemic agency. The chapter is divided into two sections,

with the first elaborating on the unit of analysis, and the second elaborating on participants' interaction as they enacted the innovative pedagogy. The first section discussed the six characteristics that encapsulate the shared epistemic agency that this study seeks to awaken amongst the participants. Analysis of the first characteristic, Intention, showed that Intentions orient toward the knowledge-building processes of Extension, Explication, or Expertise, and are triggered by a participant's desire to resolve an Assumed or Identified unknowing. The unknowing could be the participant's own, or that of another participant or group of participants.

The findings from the second part of this section, which discussed knowledgebuilding practices, showed how each of the characteristics was more nuanced in their enactment than is suggested by the literature. These nuanced depictions of the characteristics – which reveal the modes of Extension, Explication, and Expertise – showed how the participants operationalised the characteristics as they enacted the innovative pedagogy. The different qualities and values of Mutual Relations were also discussed. Analysis of the third part of an Episode led to the determination of the end of an Episode, and of the different ways of resolving unknowing that resulted in the achievement of New Knowledge.

The second section of this chapter addressed on participant interaction, and highlighted how positioning, process authority, and epistemic authority and their interactions were evidenced in the classroom. It gave an

indication of the active participation and participants relationship with their mathematics as was the aim of the study.In particular, the participants' process authority was seen to emerge as the result of a blending of authority, which itself arose from the mutual interdependency of participants' experiences and skills. Command over dialogical and physical action was dependent on agency of participants in their project of advancing their individual and collective mathematics knowledge, and Extension emerged as an unexpected means of controlling and managing the behaviour of others.

As participants interacted to enact the innovative pedagogy, their relationships to epistemic authority revealed that certain background knowledge was required in order for a participant to direct their agency towards Extension. Moreover, it was determined that participants enacted the pedagogy regardless of the ability labels assigned to them by the school; as a group, participants took responsibility for their individual and collective knowledge advancement. In the following chapter, I will explicitly apply these findings to the research questions.